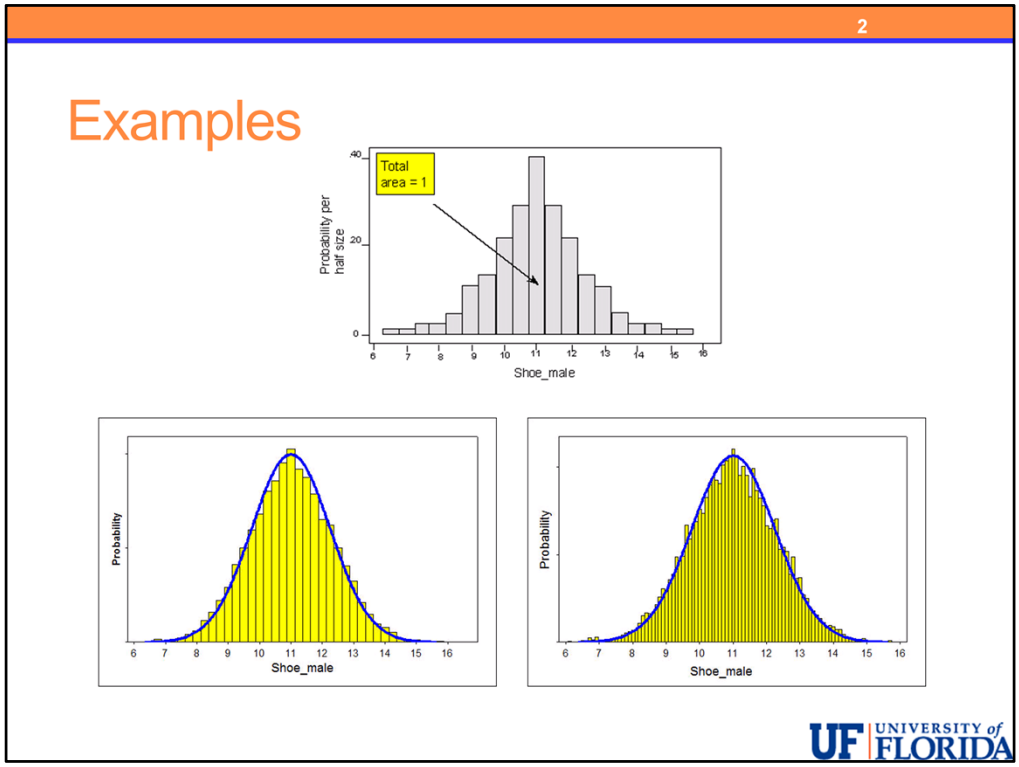
CONTINUOUS RANDOM VARIABLES

Unit 3B: Random Variables

Now we turn our attention to continuous random variables.

We will begin with the important concepts which apply to all continuous random variables in general and then we will learn to work with the special case of normally distributed random variables.



We already stated that continuous RVs can take on **ANY** value in an interval. The interval can, at least theoretically, range from negative infinity to positive infinity.

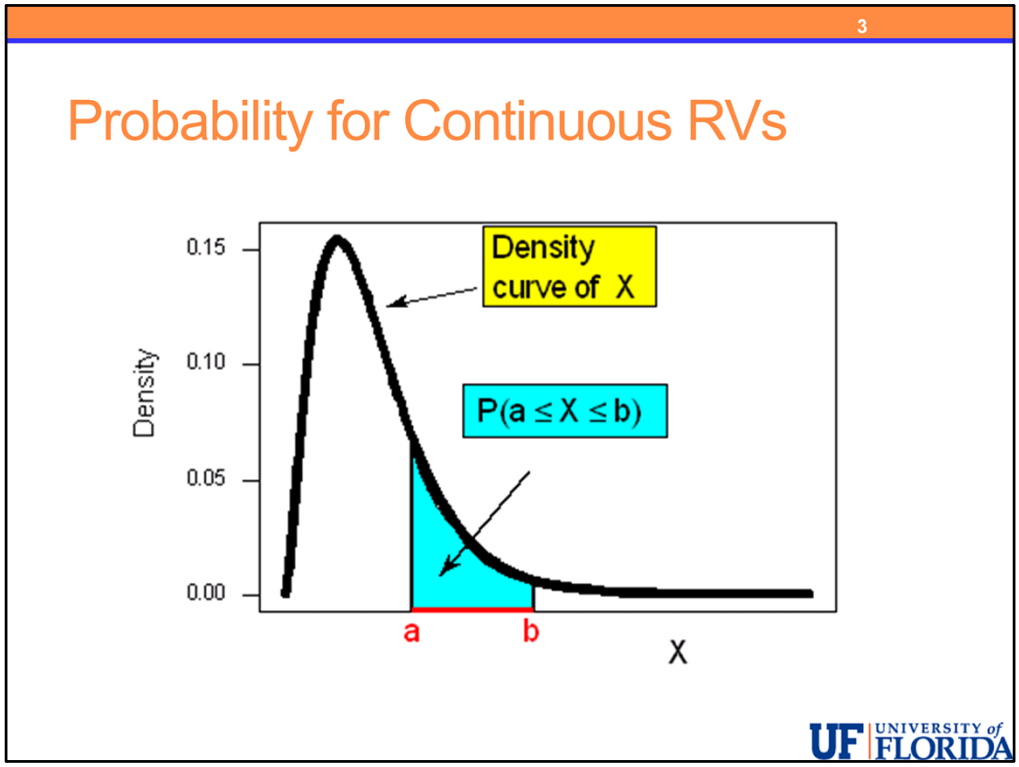
Because we cannot list each value, the probability function, $P(X = x)$ used for discrete distributions cannot be used.

Here we illustrate what happens when our intervals become increasingly small. The area of each rectangle will also become increasingly small.

For a continuous random variable, we can no longer add rectangles but must find the area under the curve.

This curve is called the probability density function.

The underlying mathematics requires calculus and so we will rely on tables or online calculators to do the required calculations.



For continuous RVs, $P(X = x) = 0$.

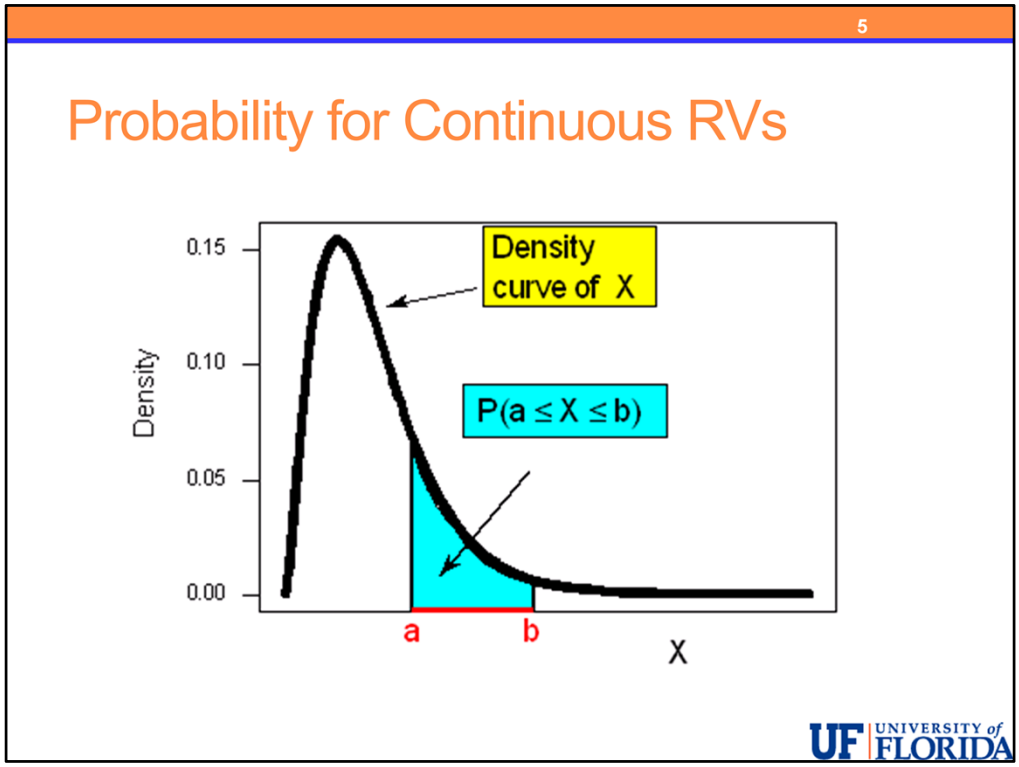
However $P(a < X < b)$ makes sense and will be found as the area under the probability density curve for that random variable.

Continuous Random Variables

- For continuous RVs, $P(X = x) = 0$!!!
- However $P(a < X < b)$ makes sense
  - A person's height is 62" – what does that REALLY mean?
  - Person's height ROUNDS to 62"
  - Person's height is greater than or equal to 61.5 inches but less than 62.5 inches.
  - Or P(61.5 ≤ Height < 62.5)

Let's think a little about this.

- If I tell you a person's height is 62" what does that REALLY mean?
- For measured variables such as this in research, it will mean that the person's height ROUNDS to 62"
- Which will mean that the person's height is greater than or equal to 61.5 inches but less than 62.5 inches.
- So we could find P(61.5 ≤ Height < 62.5)

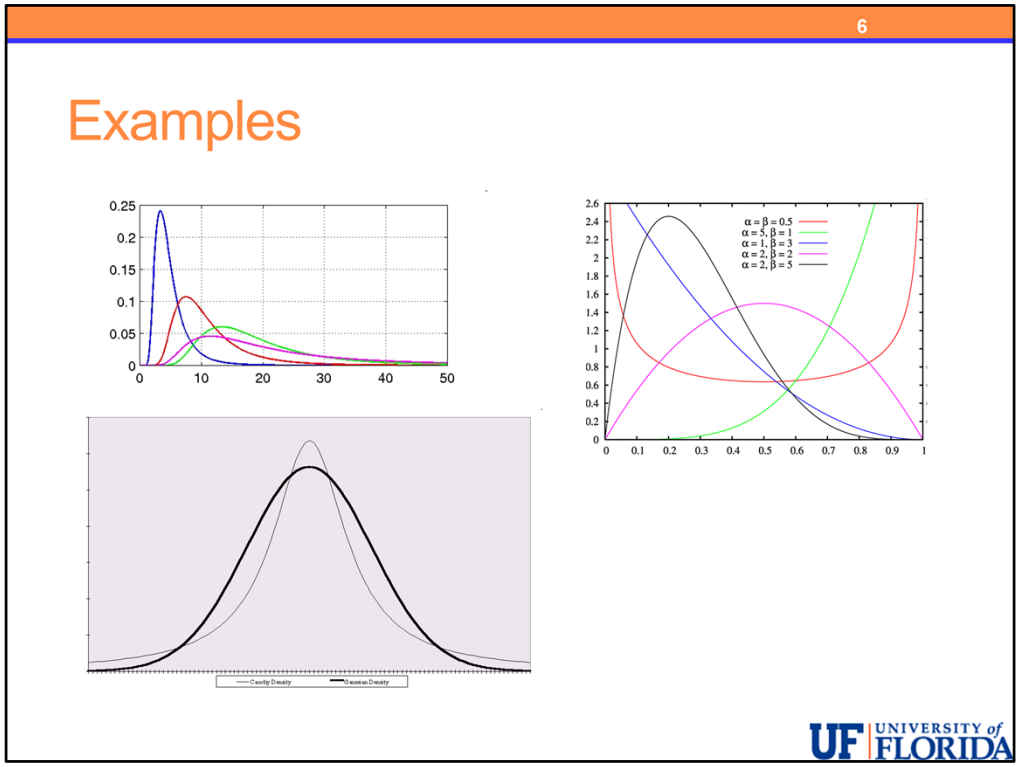


The area between a and b under the PDF is equal to $P(a < X < b)$.

For continuous random variables it does not matter whether we include the equals in our probabilities, since $P(X = x) = 0$.

Be careful as this distinction remains important for discrete random variables which you may see again.

Finally, it is also important to note that the total area under the probability density function is equal to 1.



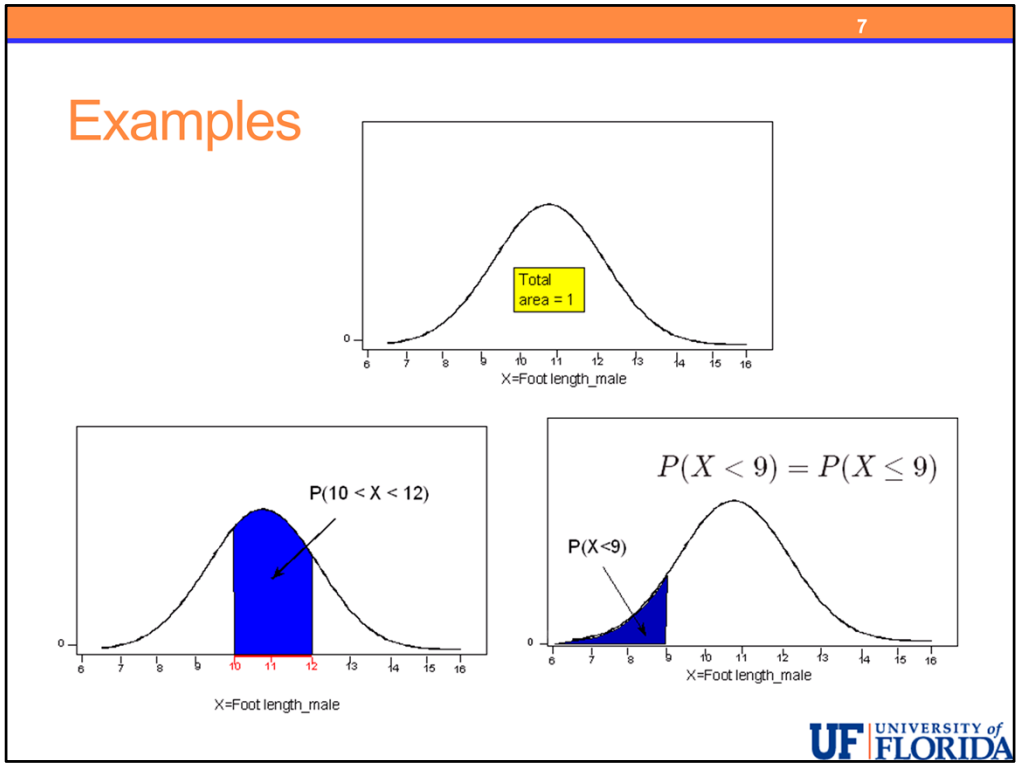
Any shape is possible in order to model all types of real-world situations.

The probability density function has

Large values in areas of high probability

Small values in regions of low probability

HOWEVER, value of the probability density function, the height, is not $P(X = x)$



Here are a few examples like those we will be working soon.

We will learn to calculate probabilities and work backwards from probabilities to find scores.

In these problems it is good to sketch the situation and the area of interest.

Notice again on the lower right that to find the P(X < 9) is the same as finding P(X ≤ 9) since P(X = 9) = 0.

Measures for Continuous RVs

- Expected Value = average value of the RV

$$E(X) = \int_{-\infty}^{\infty} x f(x) dx = \mu$$

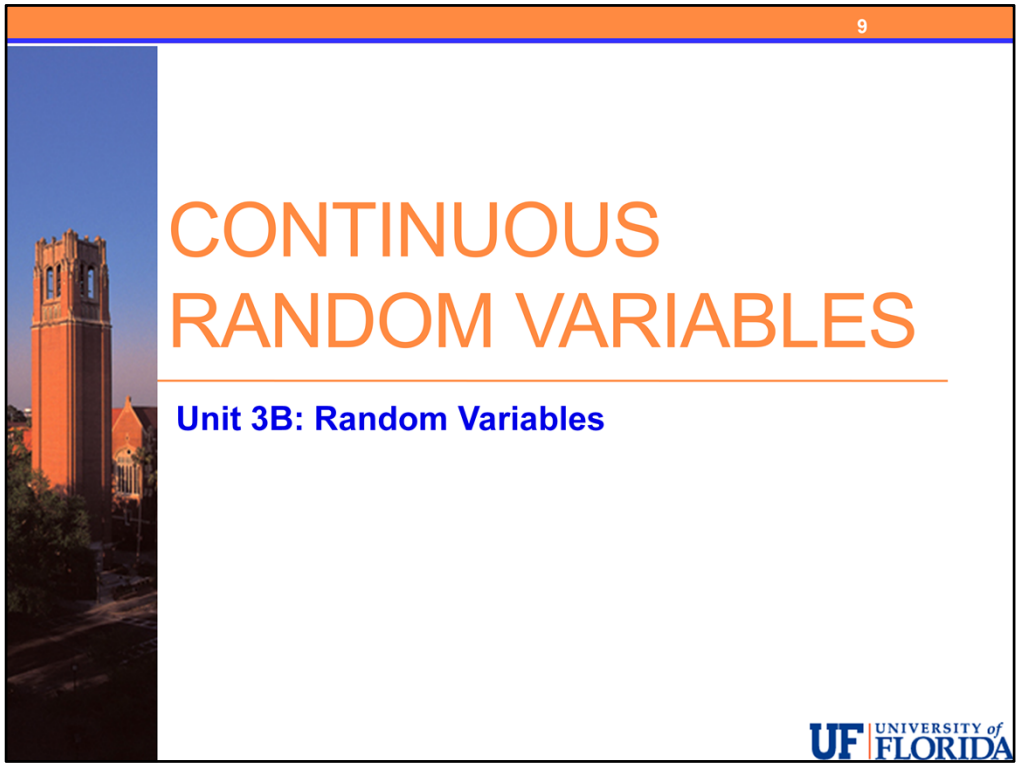
- Variance

$$Var(X) = E[(X-\mu)^2] = \sigma^2$$

- Requires Calculus to contemplate mathematics

Similar to discrete random variables, there are equations for the mean, variance, and standard deviation.

However, these equations require calculus and we will not be working with them in this course.



Continuous random variables require a different approach than discrete random variables.

We must find the area under the curve in order to calculate probabilities.

Now we will focus on the normal distribution.