

STANDARD NORMAL

Unit 3B: Random Variables



The standard normal distribution is an important special case which has a mean of zero and a standard deviation of 1.

Standard Normal Distribution

- Mean = 0 and Variance = 1
 - $\approx 68\%$ of the area lies between +1 and -1
 - $\approx 95\%$ of the area lies between +2 and -2
 - $\approx 99\%$ of the area lies between +3 and -3
- The previous approximations are often termed the “Empirical Rule” we used “Standard Deviation Rule”
- Normal Probability Tables or Online Calculators can be used to find probabilities and z-scores

Thinking back to the standard deviation rule, since the standard deviation is equal to one, we can say that

- $\approx 68\%$ of the area lies between +1 and -1
- $\approx 95\%$ of the area lies between +2 and -2
- $\approx 99\%$ of the area lies between +3 and -3

Now we want to find the exact probabilities using normal tables or online calculators.

Normal Distributions

- Converting from $X \Leftrightarrow Z$

$$Z = \frac{x - \mu}{\sigma} \qquad X = \mu + z\sigma$$

Any normal distribution can be converted to a standard normal and vice versa using these equations.

On the left, we have the conversion from a general normal random variable X to a standard normal random variable Z using an equation similar to the z-score equation from Unit 1 except that we have the population mean and population standard deviation instead of those from a sample.

On the right, we have the conversion from a z-score back to a general normal random variable X with mean μ and standard deviation σ .

Normal distributions are commonly used in statistics both as approximations for our sample, and as we will soon see an approximation of the distribution of a statistic from a sample.

They can also be used to approximate some discrete distributions in certain circumstances – although this will not be our focus.

Tables and Online Calculators

- We now demonstrate a few methods for calculating normal probabilities
- Table:
<http://media.news.health.ufl.edu/misc/bolt/Intro/PHC6050-6052/00Documents/normal.pdf>
- Online Calculator – Version 1 (JAVA):
<http://www.stat.tamu.edu/~west/applets/normaldemo2.html>
- Online Calculator – Version 2 (Non-JAVA):
<http://homepage.stat.uiowa.edu/~mbognar/applets/normal.html>



The links to the normal table and online calculators are provided in the online materials.

Table:

<http://media.news.health.ufl.edu/misc/bolt/Intro/PHC6050-6052/00Documents/normal.pdf>

Online Calculator – Version 1 (JAVA):

<http://www.stat.tamu.edu/~west/applets/normaldemo2.html>

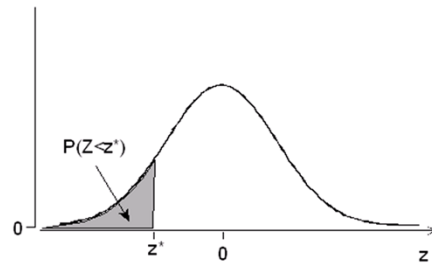
Online Calculator – Version 2 (Non-JAVA):

<http://homepage.stat.uiowa.edu/~mbognar/applets/normal.html>

Normal Table

- $P(Z < -2.81) = 0.0025$.

Standard normal probabilities			
z	.00	.01	.02
-3.4	.0003	.0003	.0003
-3.3	.0005	.0005	.0005
-3.2	.0007	.0007	.0006
-3.1	.0010	.0009	.0009
-3.0	.0013	.0013	.0013
-2.9	.0019	.0018	.0018
-2.8	.0026	.0025	.0024
-2.7	.0035	.0034	.0033
-2.6	.0047	.0045	.0044



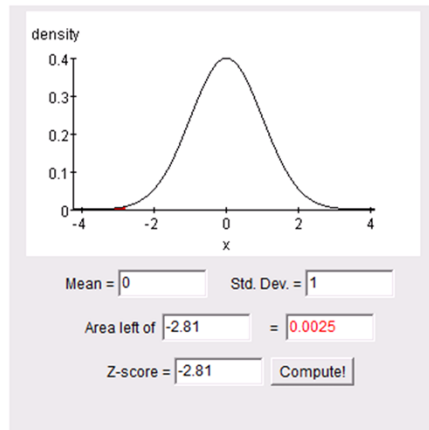
The table provides the probability $P(Z < z^*)$.

To find $P(Z < -2.81)$ we look up the first two values on the left, -2.8

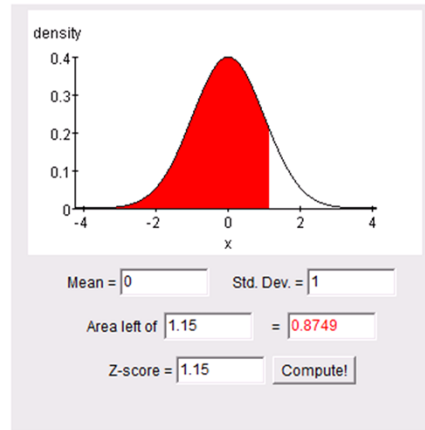
And the last digit on the top, 0.01 in this case since there is a 1 after the 8 in -2.81.

Following these values in and down until they intersect we find the probability of 0.0025.

Online Calculator – JAVA Version



$$P(Z < -2.81) = 0.0025$$



$$P(Z < 1.15) = 0.8749$$

When we developed the materials, we used the JAVA version of the online calculator, illustrated here.

Simply type in the z-score in the “Area left of” box (or the z-score box) and hit compute.

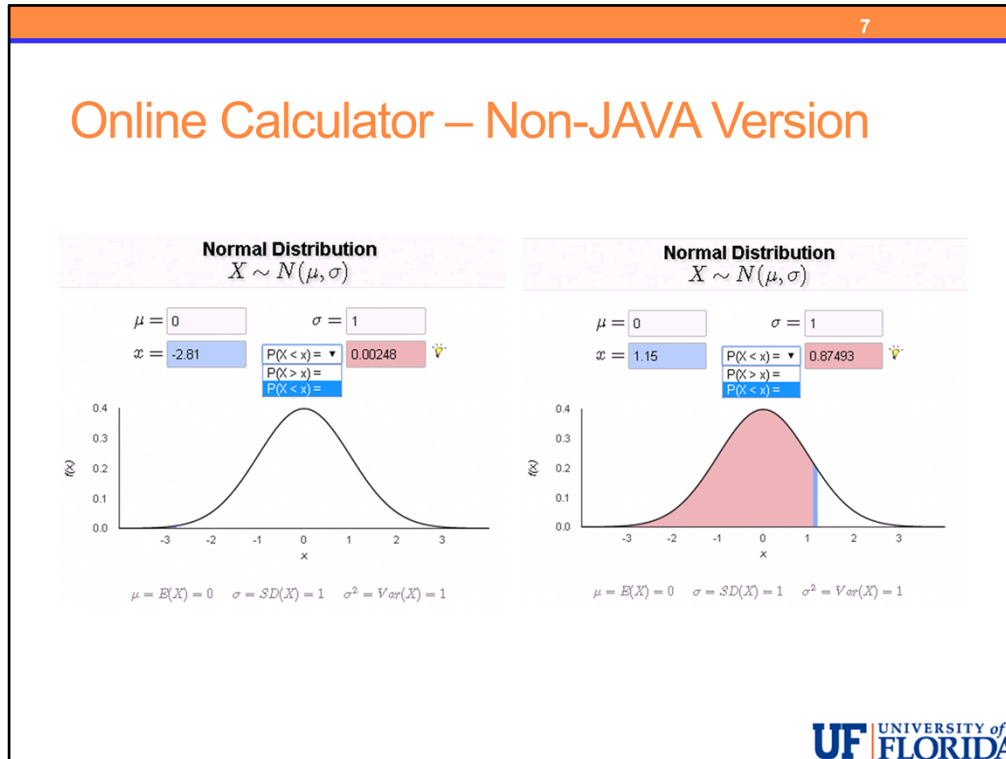
To use the calculator in reverse, simply type the probability into the box to the right of the equals with the red text and hit compute.

By default this is set up with a mean of zero and standard deviation of 1 so that we are working with the standard normal distribution.

It is our suggestion to work problems through the z-score as this is how we obtain our solutions. If you work directly with a general normal distribution by providing the mean and standard deviation to the calculator, the answers may not match exactly.

We also round all z-scores to 2 decimal places.

Online Calculator – Non-JAVA Version




Since JAVA has become somewhat painful to work with due to recent security changes, we also provide a link to this Non-JAVA version which works very similarly but also has the option to calculate probabilities above as well as below a specified value.

It can also be used in both directions and by default is set to the standard normal distribution.


There is no compute button but clicking off of the box or pressing the enter key will perform the calculations.

When working these problems it is very helpful to sketch a picture of the probability you wish to find and check it against the answer you obtain for any obvious errors.



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We use the standard normal distribution as a way to obtain the probabilities we need for more interesting applications.

We will look at an example next.