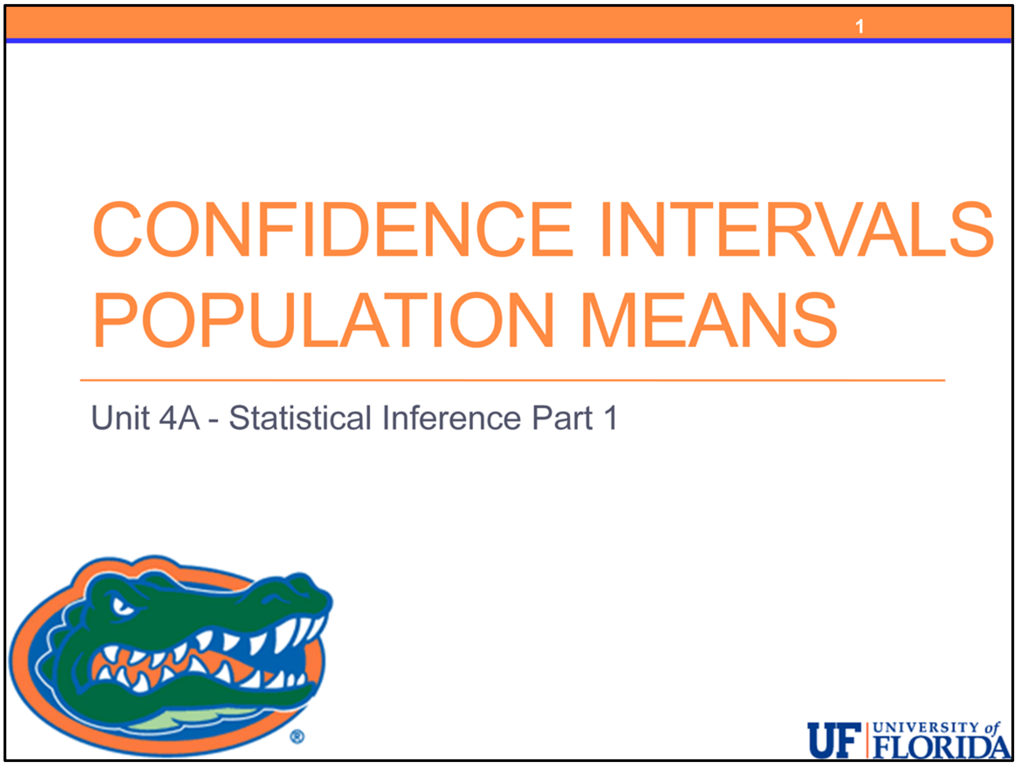
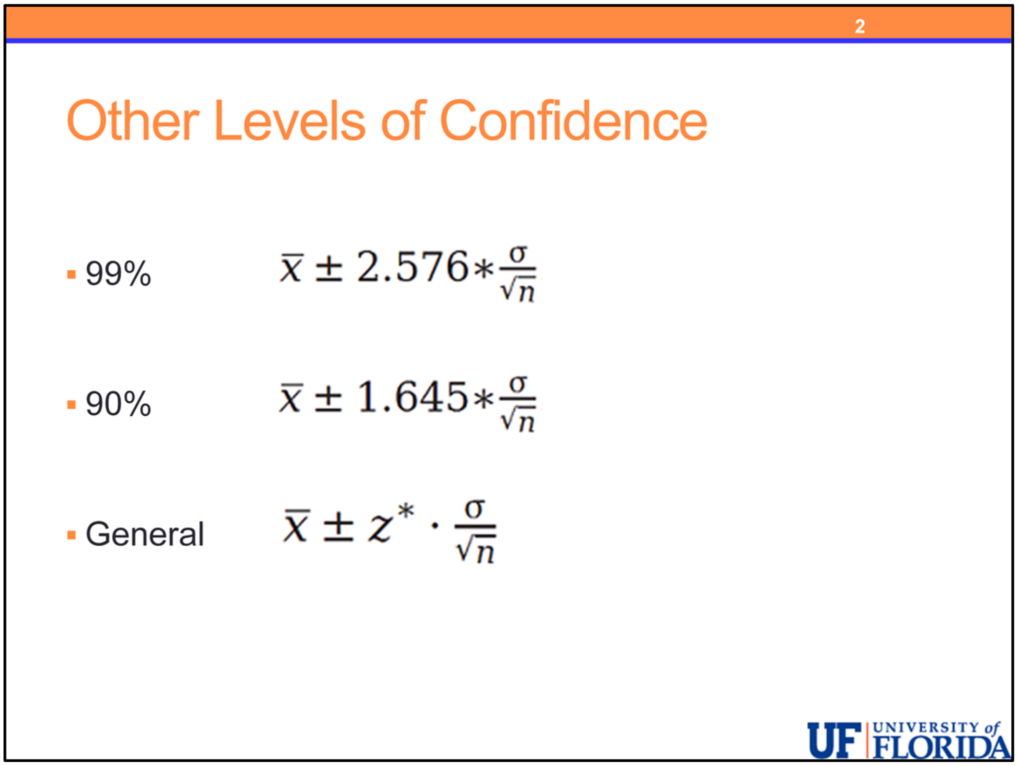


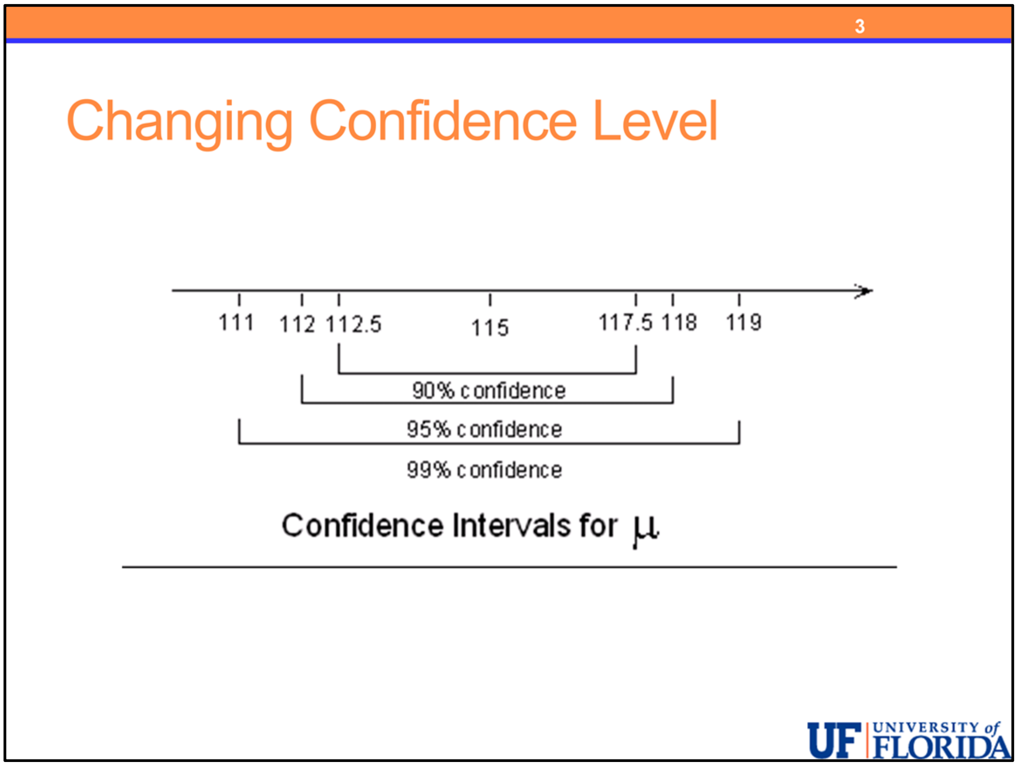
{Confidence Intervals for Population Means}

Now we will present the result for other common confidence levels and in general.



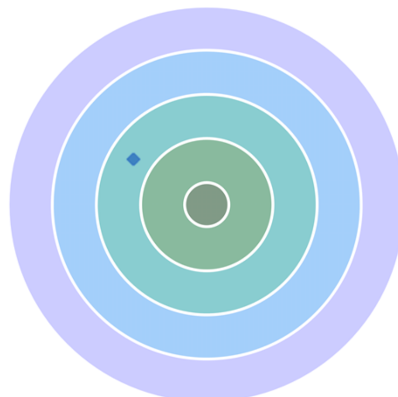
You can verify for yourself that the z-scores corresponding to the middle 99% and 90% of a standard normal distribution are 2.576 and 1.645 respectively.

In general, we use the notation z* to denote any level of confidence where we would find the needed z-score using the same approach. You should be able to find the z-score for any level of confidence we specify.



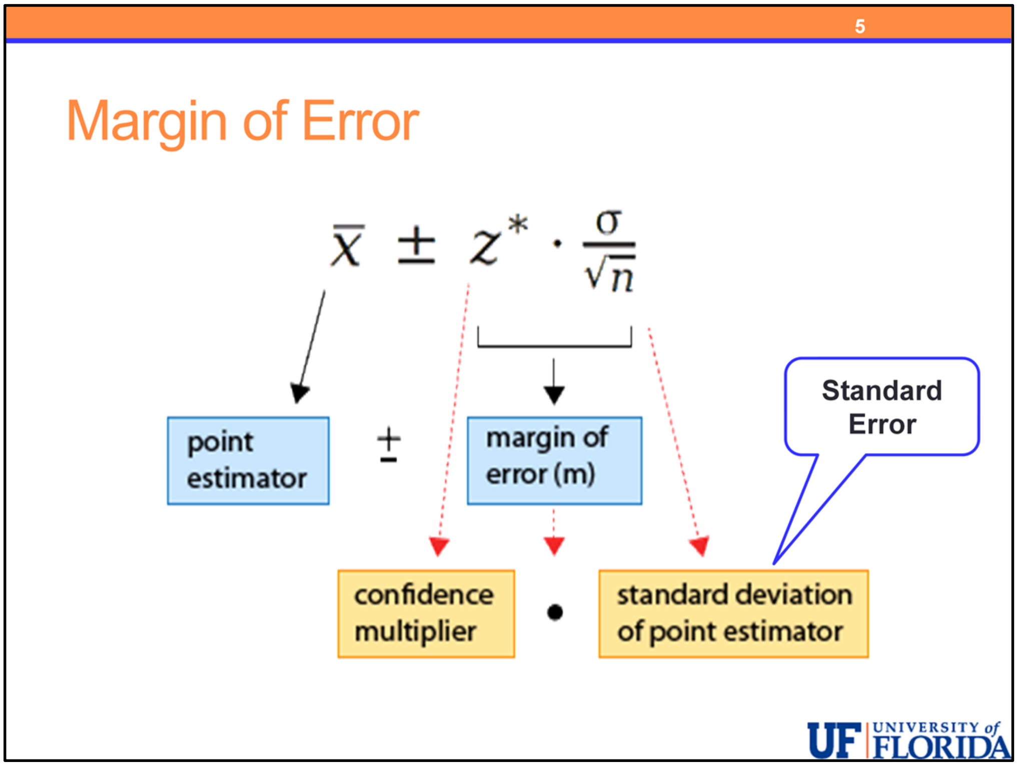
It should make sense that, all else being equal, if we increase our confidence level, the width of our interval will increase. In order to be MORE confident we must have a LARGER interval.

Dartboard Analogy



We can consider our reverse dart board analogy again.

The more confident we want to be to hit the target, the larger the dart board we throw needs to be!



There is, of course, a way to decrease the width of our confidence interval without sacrificing our level of confidence, and that is to increase the sample size. We already know that we can decrease the standard error of our statistic by increasing our sample size and the width of our confidence interval is directly related to the standard error.

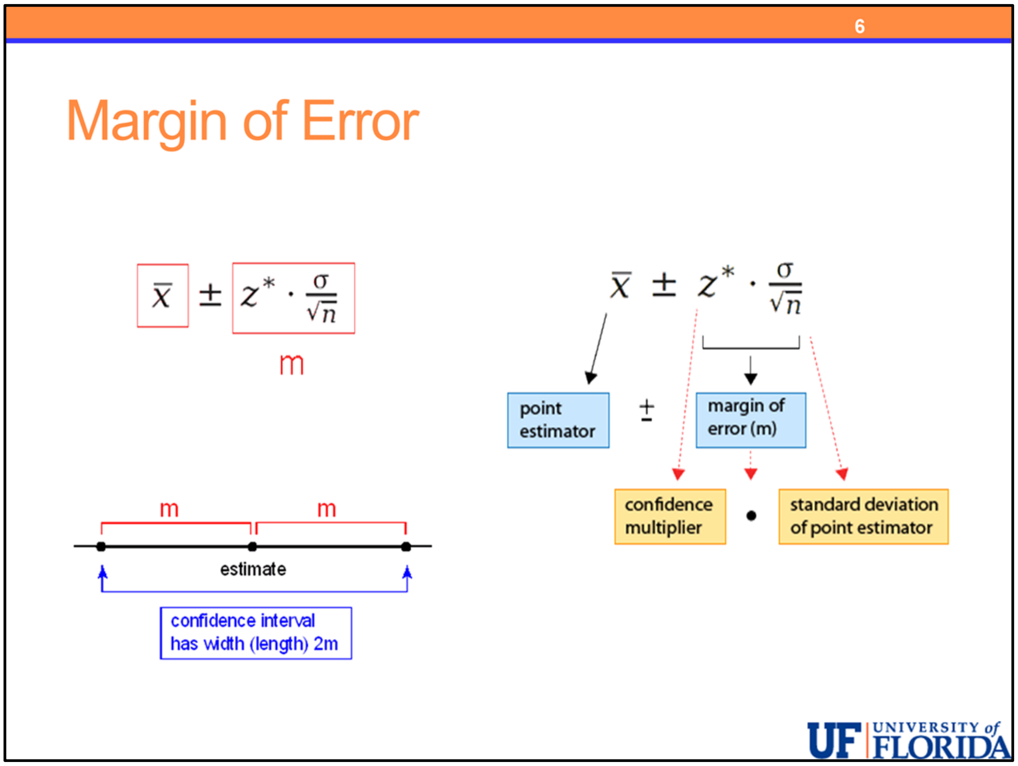
When we create our confidence interval, we add and subtract the product of the standard error and (in this case) a z-score which tells us how many standard errors we need to go on either side.

This combined result, z* times the standard error, is called the margin of error and represents the maximum estimation error for a given level of confidence.

Often we hear results reported in this way, especially in news polls. We will hear something like:

An estimated 62% of Americans are in favor of “something” with a margin of error of plus or minus 3%.

This immediately gives a confidence interval of 59% to 65%. However, notice that we don’t know the confidence level! Always look to find the confidence level in any such statement involving a “margin of error”.



Many confidence intervals have a similar form taking a point estimator plus or minus a confidence multiplier times the standard error of the point estimator (which we could also call the standard deviation of the point estimator but more commonly this would be called the standard error.)

Don't confuse the margin of error with the standard error. The standard error is only a portion of the formula for the margin of error.

Also notice that the confidence interval has a width that it is twice the margin of error or alternatively, the margin of error is half the width of the confidence interval.

The interval always contains the estimate from our sample, in fact it is ALWAYS the case that the point estimate from our data will be at the CENTER of the confidence interval.

As we can see from the equation for the margin of error, if we increase the sample size, we will decrease the margin of error and hence the width of our confidence interval.

You should work through some examples yourself to see this in action!

We have now seen the theoretical development of confidence intervals for one population mean. We have seen that it is based upon the results of the sampling distribution of x-bar and therefore how probability and probability distributions are an underlying aspect of confidence intervals.

Calculating confidence intervals for a population mean requires knowledge of the population standard deviation and the ability to approximate the sampling distribution by a normal distribution.

Then we need the sample size and sample mean from our data.

Once we have our result, we must be careful to interpret it accurately in the terms of our scenario.