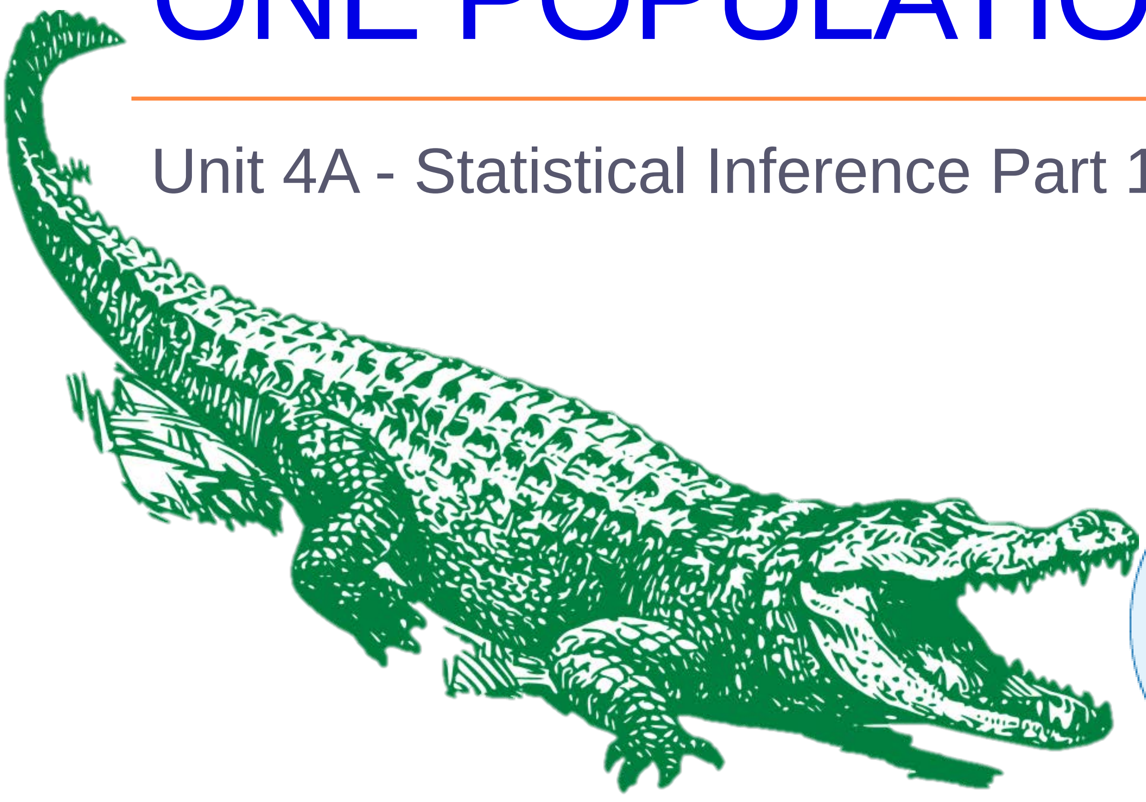
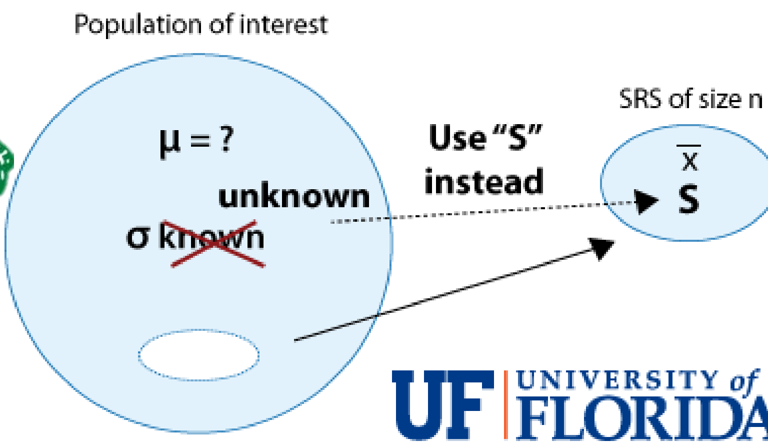
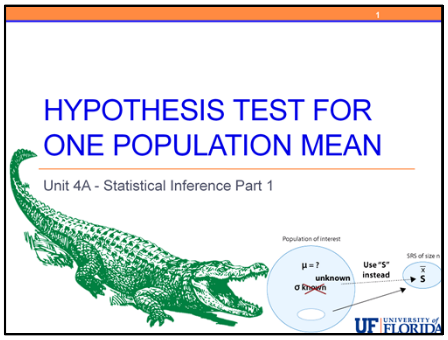
HYPOTHESIS TEST FOR ONE POPULATION MEAN

Unit 4A - Statistical Inference Part 1







Now we will look at the one-sample t-test for a population mean.

When we discussed confidence intervals we looked at both the z-based confidence interval where we assumed we knew the population standard deviation and the t-based confidence interval where we only had access to the sample standard deviation, s.

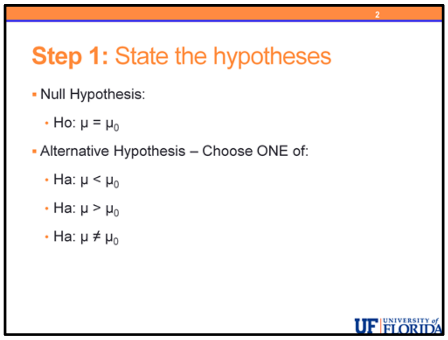
We could go through the same two situations here but in practice it is extremely rare to know the population standard deviation and so we will only look at the t-test where we will use the sample standard deviation as our estimate of the population standard deviation.

This substitution results in the need to use a t-distribution for p-values and cutoffs for confidence intervals.

We used the z-based results to give us good foundational examples we could easily work by hand. From now on, we rely on software to find the needed p-values and confidence intervals for us. We will focus on correctly using and interpreting these results.

Step 1: State the hypotheses

- Null Hypothesis:
  - Ho: $\mu = \mu_0$
- Alternative Hypothesis – Choose ONE of:
  - Ha: $\mu < \mu_0$
  - Ha: $\mu > \mu_0$
  - Ha: $\mu \neq \mu_0$



In STEP 1, we set up our hypotheses. This is almost identical to that for one population proportion except that the notation changes to reflect that we are now talking about a population mean, μ (mu).

Our null hypothesis will always be the EQUALS with Ho: $\mu = \mu_0$

And our alternative hypothesis can take on one of three forms. Either we have

- Ha: $\mu < \mu_0$
- Ha: $\mu > \mu_0$ OR
- Ha: $\mu \neq \mu_0$

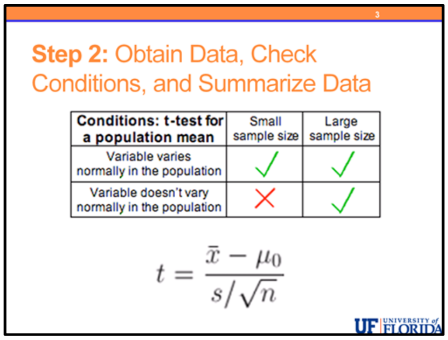
The first two of these choices are one-sided tests and the last is a two-sided test. From this point we will rarely conduct one-sided tests in this course but we will still briefly discuss the concepts here.

When setting up hypotheses, be sure to NOT use any information about the sample data collected to help you choose the correct alternative hypothesis. Use only the information about the research question of interest to make this choice.

Step 2: Obtain Data, Check Conditions, and Summarize Data

Conditions: t-test for a population mean	Small sample size	Large sample size
Variable varies normally in the population	✓	✓
Variable doesn't vary normally in the population	✗	✓

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$$



First we would OBTAIN our DATA: In this step we would **obtain data from a sample.** This is not something we do much of in courses but it is done very often in practice!

Then we need to CHECK that the CONDITIONS to use the test are satisfied - which are:

- The **sample is random** (or at least can be considered random in context).
- **We are in one of the three situations marked with a green check mark** in the table (This ensures that x-bar is at least approximately normal and the test statistic using the sample standard deviation, s, is therefore a *t*-distribution with n-1 degrees of freedom – proving that the test statistic follows a t-distribution is well beyond the scope of this course but it is a fact we need to use).

The result is that:

- **For large samples,** we **don't need to check for normality in the population**. We can rely on the sample size as the basis for the validity of using this test.
- **For small samples**, we **need to have data from a normal population** in order for the p-values and confidence intervals to be valid.

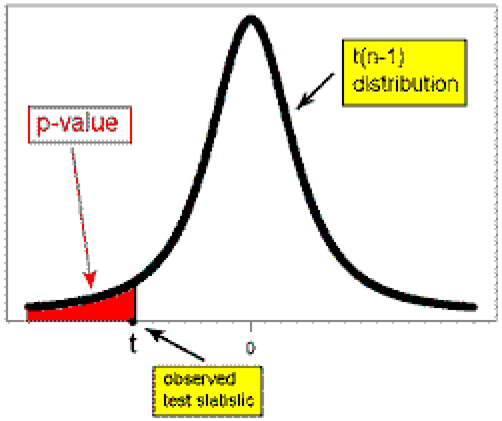
There are a number of activities in the materials related to checking for normality for small samples as well as an overall summary of when we can apply this test. Please go through these activities carefully to get a better sense of this step in practice.

If the conditions are satisfied then we calculate the test statistic which is now a "t-score" instead of a "z-score".

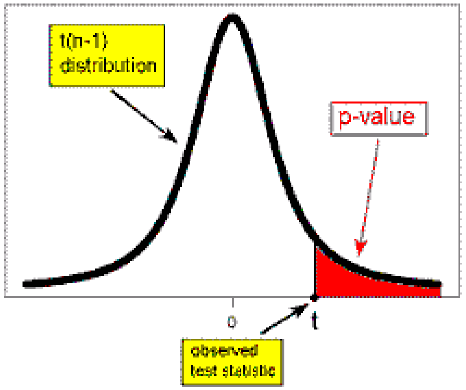
- The only difference is that now it only provides an **estimate** of the number of standard errors our data fall above or below the null value and this estimation (caused by using the sample standard deviation to estimate the true population standard deviation) requires us to use the t-distribution to calculate p-values for this test.
- That is about as deep as we want to go into the theory behind this test.
- Although we will rely on software to calculate p-values, it is still easy enough to calculate the test statistic by hand if we have the sample mean, sample standard deviation, and sample size.

Step 3: Find the P-VALUE of the Test

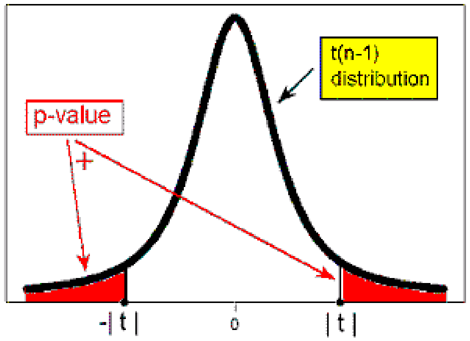
Ha: $\mu < \mu_0$



Ha: $\mu > \mu_0$



Ha: $\mu \neq \mu_0$



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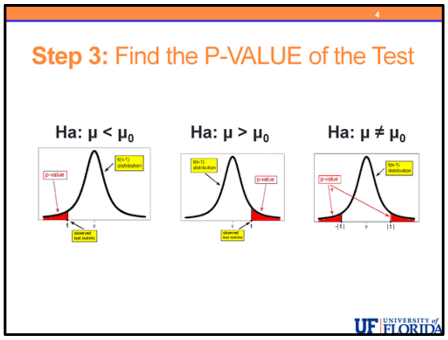
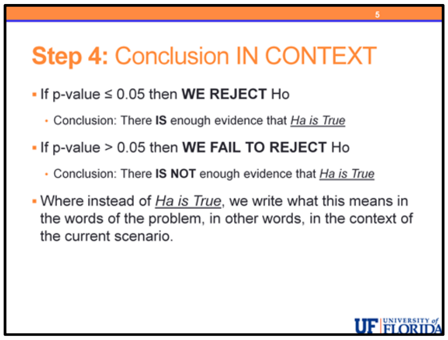


We won't be finding the p-value by hand but the idea is exactly the same. We need to find probabilities to the left for less than tests, to the right for greater than tests, probabilities on both tails for not-equal-to tests.

Step 4: Conclusion IN CONTEXT

- If p-value ≤ 0.05 then **WE REJECT** Ho
  - Conclusion: There **IS** enough evidence that *Ha is True*
- If p-value > 0.05 then **WE FAIL TO REJECT** Ho
  - Conclusion: There **IS NOT** enough evidence that *Ha is True*
- Where instead of *Ha is True*, we write what this means in the words of the problem, in other words, in the context of the current scenario.



There is nothing new in this step except that in context, we will be discussing population means instead of population proportions.

Let's review what we said before.

If p-value ≤ 0.05 then **WE REJECT** Ho
- Conclusion: There **IS** enough evidence that ***<u>Ha is True</u>***

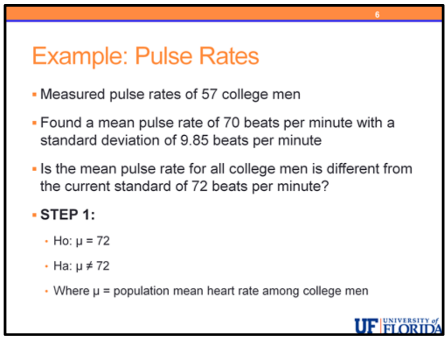
If p-value > 0.05 then **WE FAIL TO REJECT** Ho
- Conclusion: There **IS NOT** enough evidence that ***<u>Ha is True</u>***

Where instead of *<u>Ha is True</u>*, we write what this means in the words of the problem, in other words, in the context of the current scenario.

This step has essentially two sub-steps:
(i) Based on the p-value, **determine** whether or not the results are **significant** (i.e., the data present enough evidence to reject Ho).
(ii) State your **conclusions** in the **context** of the problem.

Example: Pulse Rates

- Measured pulse rates of 57 college men
- Found a mean pulse rate of 70 beats per minute with a standard deviation of 9.85 beats per minute
- Is the mean pulse rate for all college men is different from the current standard of 72 beats per minute?
- **STEP 1:**
  - Ho: $\mu = 72$
  - Ha: $\mu \neq 72$
  - Where μ = population mean heart rate among college men



A research study measured the pulse rates of 57 college men and found a mean pulse rate of 70 beats per minute with a standard deviation of 9.85 beats per minute.

Researchers want to know if the mean pulse rate for all college men is different from the current standard of 72 beats per minute.

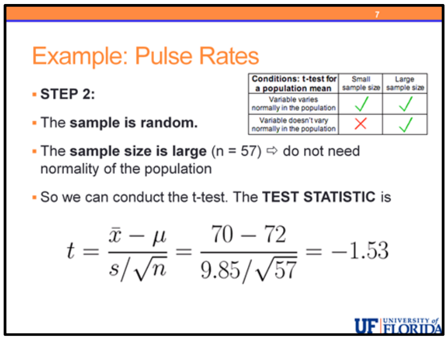
The null and alternative hypotheses in this case are:
- Ho: $\mu = 72$
- Ha: $\mu \neq 72$
- Where μ = population mean heart rate among college men

Example: Pulse Rates

Conditions: t-test for a population mean	Small sample size	Large sample size
Variable varies normally in the population	✓	✓
Variable doesn't vary normally in the population	✗	✓

- **STEP 2:**
- The **sample is random.**
- The **sample size is large** (n = 57) ⇨ do not need normality of the population
- So we can conduct the t-test. The **TEST STATISTIC** is

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{70 - 72}{9.85/\sqrt{57}} = -1.53$$



Now we need to check our conditions and if they are satisfied, calculate our test statistic.

We are told the **sample is random.**

And, the **sample size is large** (n = 57) so we do not need normality of the population in order to be able to conduct this test for the population mean. We are in the 2nd column in the table.

So we can conduct the t-test. Now we need to calculate the test statistic which has in the numerator, our sample mean of 70 minus the null value of 72 and in the denominator we have the sample standard deviation of 9.85 divided by the square root of n which is 57.

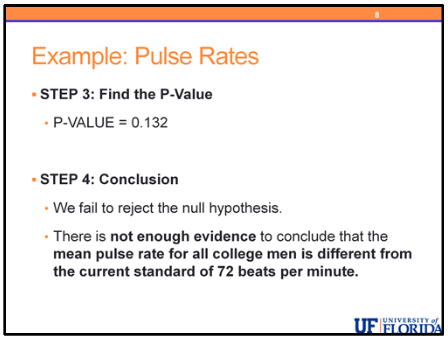
This results in a test statistic of t = -1.53.

The data (represented by the sample mean) are 1.53 estimated standard errors below the null value.

Example: Pulse Rates

- **STEP 3: Find the P-Value**
  - P-VALUE = 0.132

- **STEP 4: Conclusion**
  - We fail to reject the null hypothesis.
  - There is **not enough evidence** to conclude that the **mean pulse rate for all college men is different from the current standard of 72 beats per minute.**



Now for STEP 3.

The p-value is calculated using the t-distribution with n-1 degrees of freedom. Since n is 57, the distribution used would have 56 degrees of freedom. This is simply another parameter used for the t-distribution and you do not need to know anything more about degrees of freedom.

Using an online calculator, we found the p-value to be 0.132.

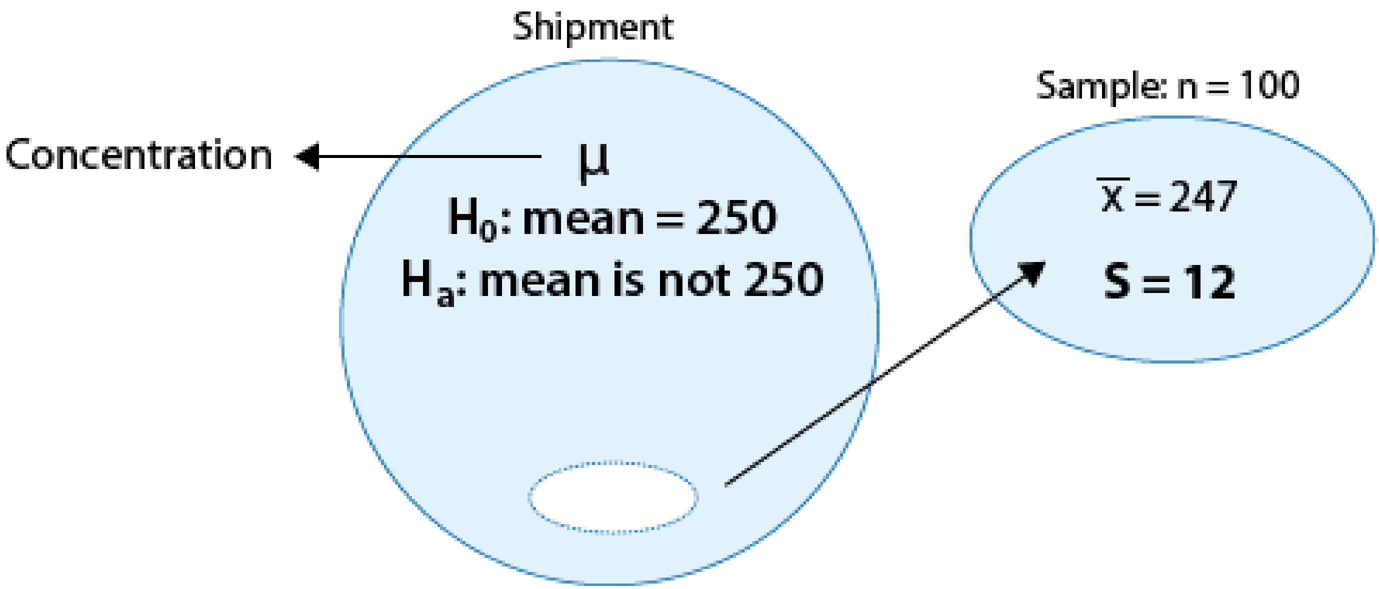
Finally, STEP 4.

The p-value (0.132) is not small, indicating that the results are not significant. We fail to reject the null hypothesis.

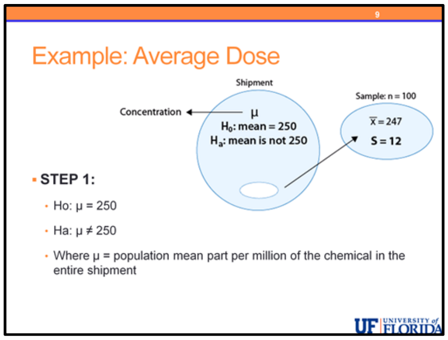
STATING OUR CONCLUSION IN CONTEXT WE SAY:

- There is **not enough evidence** to conclude that the **mean pulse rate for all college men is different from the current standard of 72 beats per minute.**

Example: Average Dose



- **STEP 1:**
  - Ho: μ = 250
  - Ha: μ ≠ 250
  - Where μ = population mean part per million of the chemical in the entire shipment



A certain prescription medicine is supposed to contain an average of 250 parts per million (ppm) of a certain chemical. If the concentration is higher than this, the drug may cause harmful side effects; if it is lower, the drug may be ineffective.

The manufacturer runs a **check to see if the mean concentration in a large shipment conforms to the target level of 250 ppm or not.**

A simple random sample of 100 portions is tested, and the sample mean concentration is found to be 247 ppm with a sample standard deviation of 12 ppm.

The hypotheses being tested are:

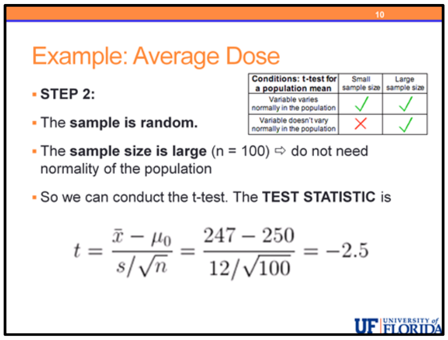
- Ho: μ = 250 (mu = mu_zero)
- Ha: μ ≠ 250 (mu ≠ mu_zero)
- Where μ = population mean parts per million of the chemical in the entire shipment

Example: Average Dose

Conditions: t-test for a population mean	Small sample size	Large sample size
Variable varies normally in the population	✓	✓
Variable doesn't vary normally in the population	✗	✓

- **STEP 2:**
- The **sample is random.**
- The **sample size is large** (n = 100) ⇨ do not need normality of the population
- So we can conduct the t-test. The **TEST STATISTIC** is

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{247 - 250}{12/\sqrt{100}} = -2.5$$



Now we need to check our conditions and if they are satisfied, calculate our test statistic.

We are told the **sample is random.**

And, the **sample size is large** (n = 100) so we do not need normality of the population in order to be able to conduct this test for the population mean. We are in the 2nd column in the table.

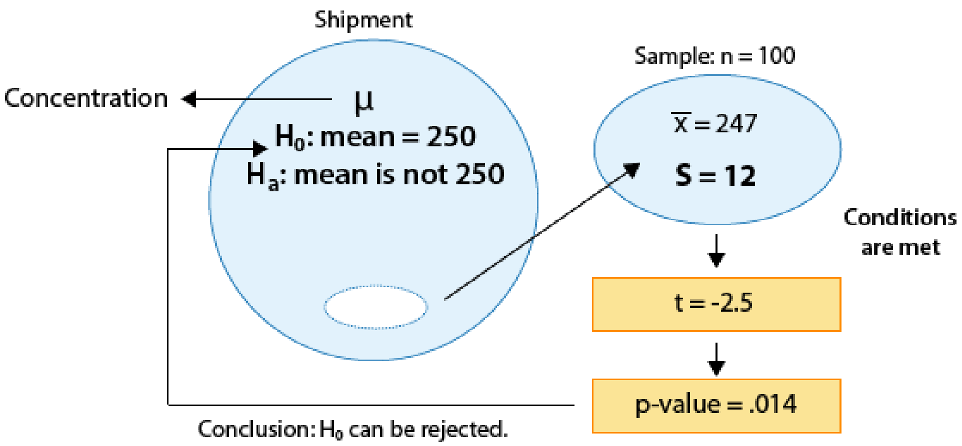
So we can conduct the t-test. Now we need to calculate the test statistic which has in the numerator, our sample mean of 247 minus the null value of 250 and in the denominator we have the sample standard deviation of 12 divided by the square root of n which is 100.

This results in a test statistic of t = -2.50.

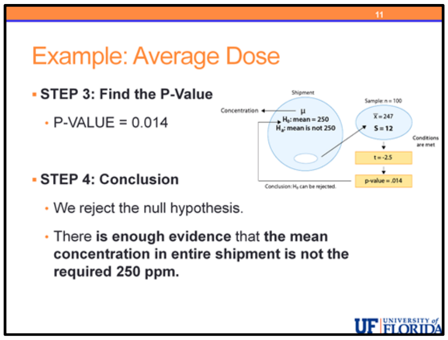
The data (represented by the sample mean) are 2.5 estimated standard errors below the null value.

Example: Average Dose

- **STEP 3: Find the P-Value**
  - P-VALUE = 0.014



- **STEP 4: Conclusion**
  - We reject the null hypothesis.
  - There **is enough evidence** that **the mean concentration in entire shipment is not the required 250 ppm.**



For STEP 3, we find the p-value to be 0.014.

And for STEP 4.

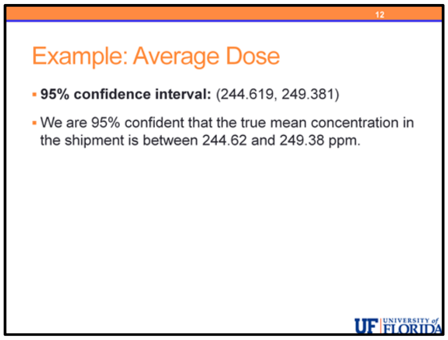
The p-value (0.014) is small, indicating that the results are significant. We reject the null hypothesis.

STATING OUR CONCLUSION IN CONTEXT WE SAY:

- There **is enough evidence** that **the mean concentration in entire shipment is not the required 250 ppm.**

Example: Average Dose

- **95% confidence interval:** (244.619, 249.381)
- We are 95% confident that the true mean concentration in the shipment is between 244.62 and 249.38 ppm.



In the same way as we discussed for proportions, the confidence interval can be used to conduct the hypothesis test – by checking to see whether or not the interval contains the null value – and adds important additional information by giving us a range of values to estimate the unknown population parameter of interest.

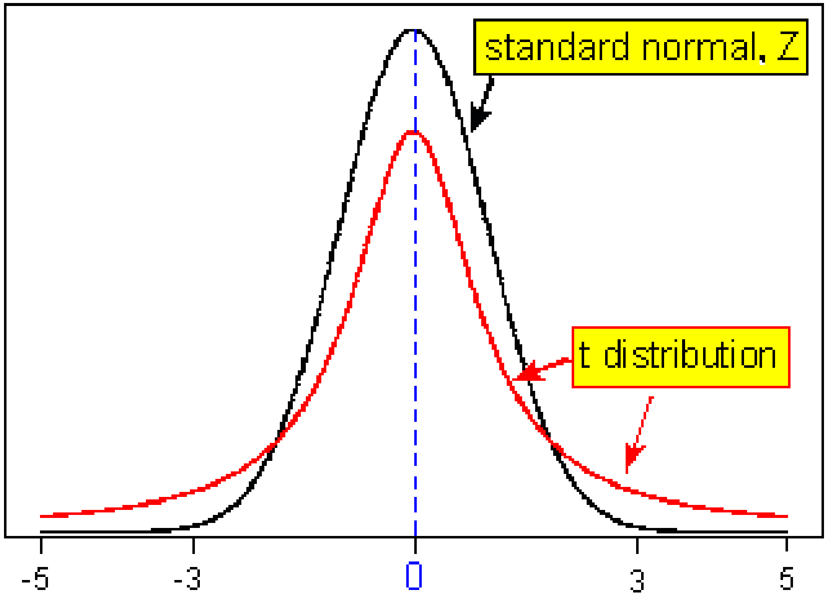
We find the **95% confidence interval to be (244.619, 249.381)**.

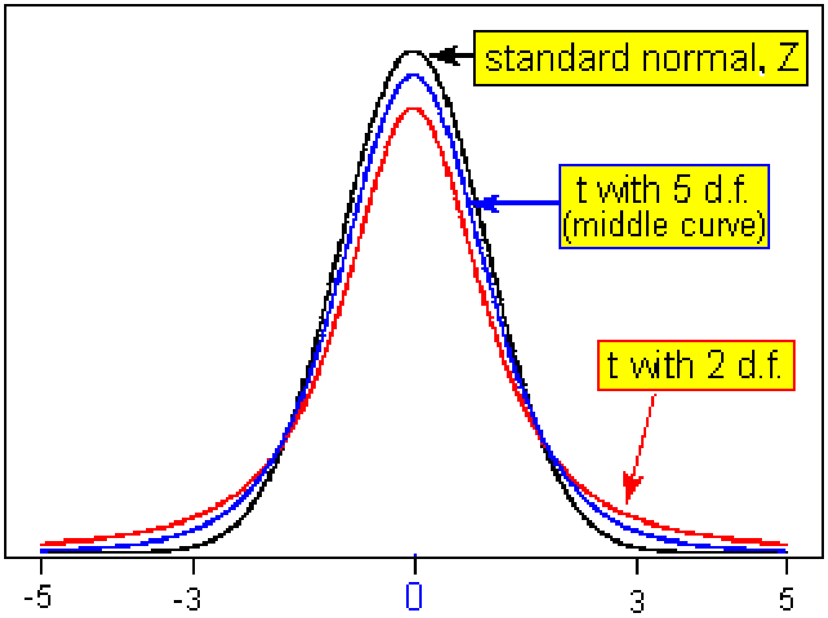
Since 250 is not in the interval we know we would reject our null hypothesis that μ (mu) = 250.

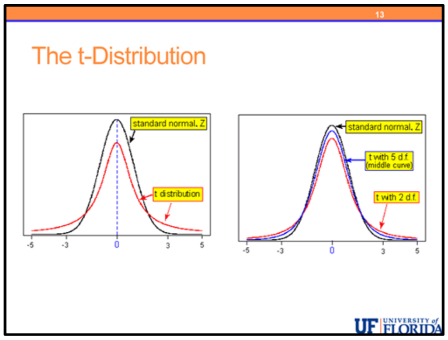
The confidence interval gives additional information. By accounting for estimation error, it estimates that the population mean is likely to be between 244.62 and 249.38. This is lower than the target concentration and that information might help determine the appropriate course of action in this situation.

We can interpret this interval by saying: We are 95% confident that the true mean concentration in the shipment is between 244.62 and 249.38 ppm.

The t-Distribution







You can learn more about the t-distribution in the materials or by searching for other sources but we will mention a few points before we end.

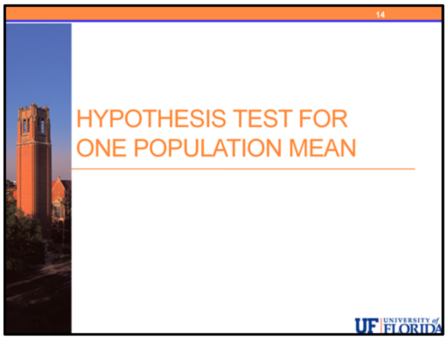
The t-distribution has heavier tails than the normal distribution. This means there is more area in the tails of the t-distribution than there would be for a standard normal distribution. This is illustrated by the image on the left. The red t-distribution clearly has more area in the tails than the black curve representing the standard normal distribution.

As a result, the t-distribution has more variation than the standard normal distribution.

There is a different t-distribution for each possible value for the degrees of freedom and, as illustrated in the image on the right, when the degrees of freedom increase the t-distribution gets closer to the standard normal distribution.

For this reason, for large samples you may sometimes see the normal distribution used regardless of whether the population standard deviation is known or not … but we will not do this in our course and most software packages will conduct a t-test unless specifically told to do otherwise.

HYPOTHESIS TEST FOR ONE POPULATION MEAN



That is the end of Unit 4A – be sure to review the procedures for confidence intervals and hypothesis tests in these two simple cases of

- One Population Proportion

And

- One Population Mean

Next we will get to the main purpose of this course and what we have been building to all semester, learning how to test for relationships between two variables.