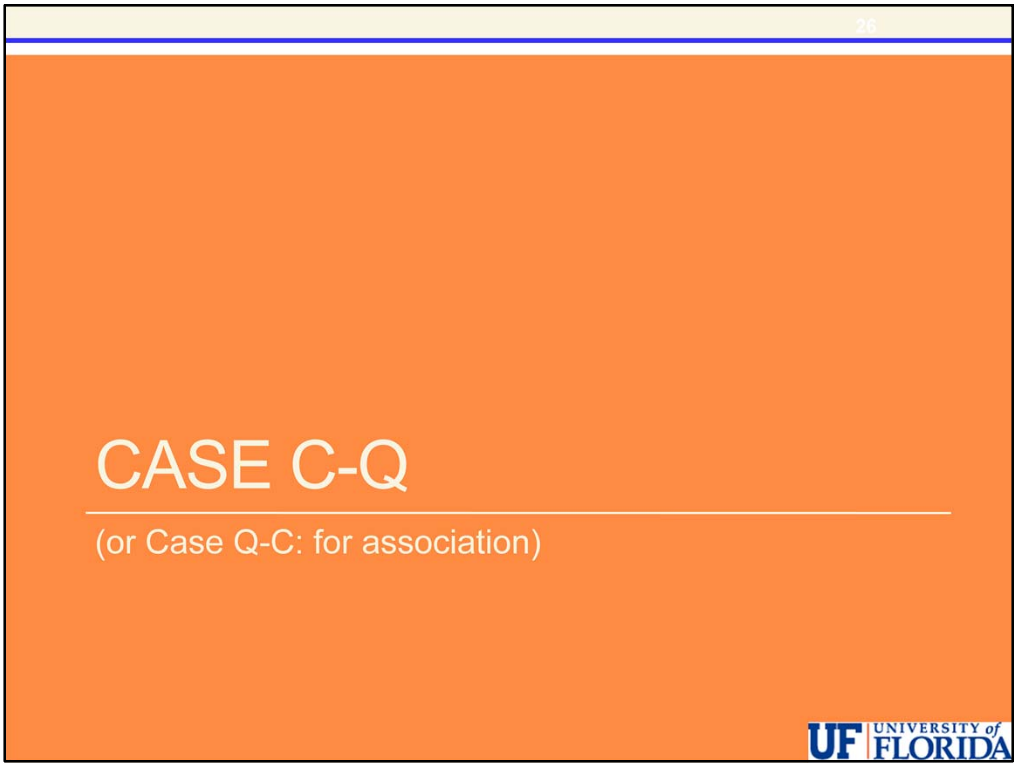


In Case C-Q we covered three main scenarios.

The first is the paired t-test which we have already reviewed.

The remaining methods are for two independent samples or for more than two independent samples.

These methods for independent samples can also be used in Case Q-C to show an association between the two variables but they will not allow us to predict a categorical outcome from a quantitative predictor as may be desired in Case Q-C.

TWO INDEPENDENT SAMPLES

Case C-Q

We will be begin with an example comparing two groups.

28

Maintaining Balance

ID	Forward/Backward	Side to Side	Age
1	21	14	Elderly
2	17	28	Elderly
3	24	21	Elderly
4	27	42	Elderly
5	24	26	Elderly
6	24	35	Elderly
7	29	23	Elderly
8	18	34	Elderly
9	31	17	Elderly
1	19	15	Young
2	16	14	Young
3	17	10	Young
4	10	7	Young
5	28	19	Young
6	30	13	Young
7	22	16	Young
8	14	10	Young

Ho: $\mu_E - \mu_Y = 0$
Ha: $\mu_E - \mu_Y \neq 0$

UF UNIVERSITY of FLORIDA

Data Reference: http://lib.stat.cmu.edu/DASL/Stories/MaintainingBalance.html

Is age related to the ability maintain balance while concentrating? The data comes from the data and story library. The data we will use was simulated to be similar to, but without some of the problems of, the original data.

Nine elderly and eight young subjects participated in this experiment.

Each subject stood barefoot on a "force platform" and was asked to maintain a stable upright position and to react as quickly as possible to an unpredictable noise by pressing a hand held button. The noise came randomly and the subject concentrated on reacting as quickly as possible.

The platform automatically measured how much each subject swayed in millimeters in both the forward/backward and the side-to-side directions.

These are two independent samples but we also have two different response variables to analyze:

- Forward to Backward Sway Range and
- Side to Side Sway Range

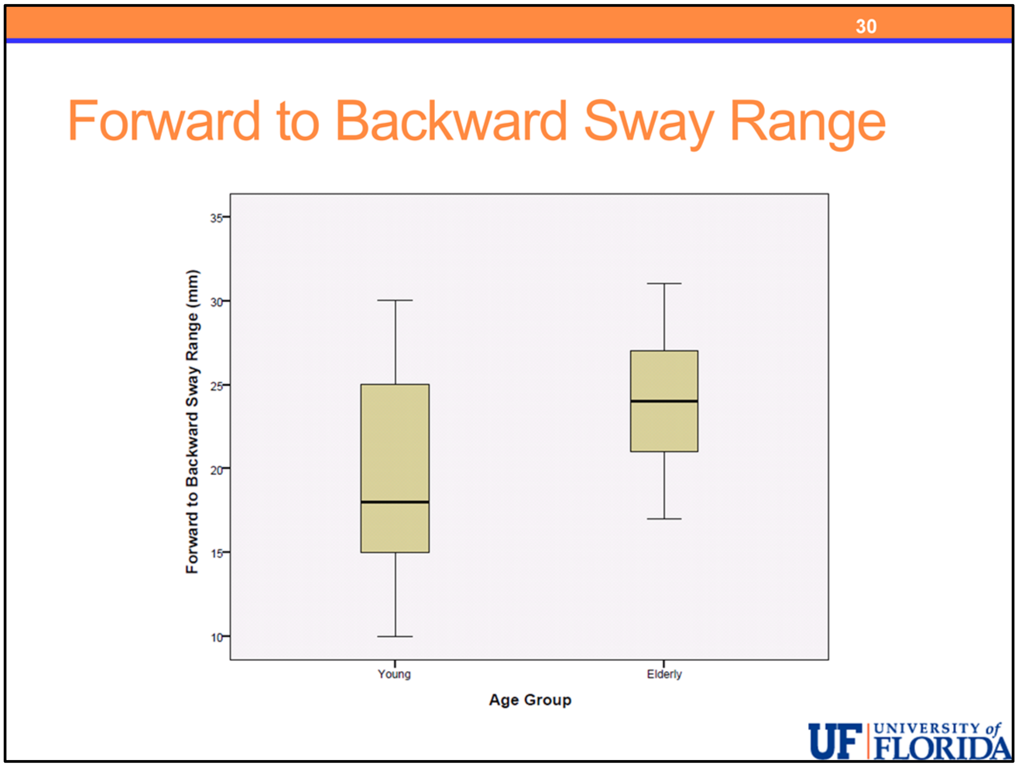
In each case our null hypotheses will be that the difference in the population mean sway range between elderly and young is zero and our alternative will be that this difference will not be equal to zero.

SPSS Summaries for Both Sway Ranges

	Age Group	N	Mean	Std. Deviation	Std. Error Mean
Forward to Backward Sway Range (mm)	Elderly	9	23.89	4.702	1.567
	Young	8	19.50	6.845	2.420
Side to Side Sway Range (mm)	Elderly	9	26.67	9.083	3.028
	Young	8	13.00	3.854	1.363

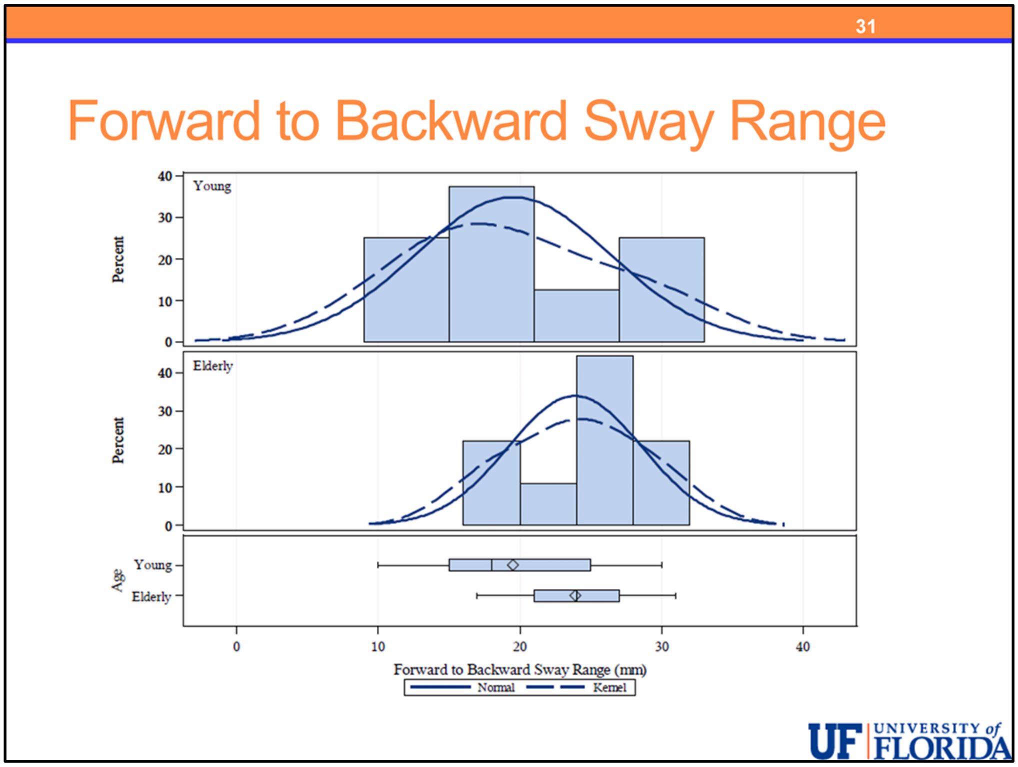
Here are the summaries produced by SPSS for both sway ranges.

Notice in SPSS the output lists elderly and then young. This indicates that the SPSS output that follows will be estimating mu-sub_Elderly minus mu-sub-Young.



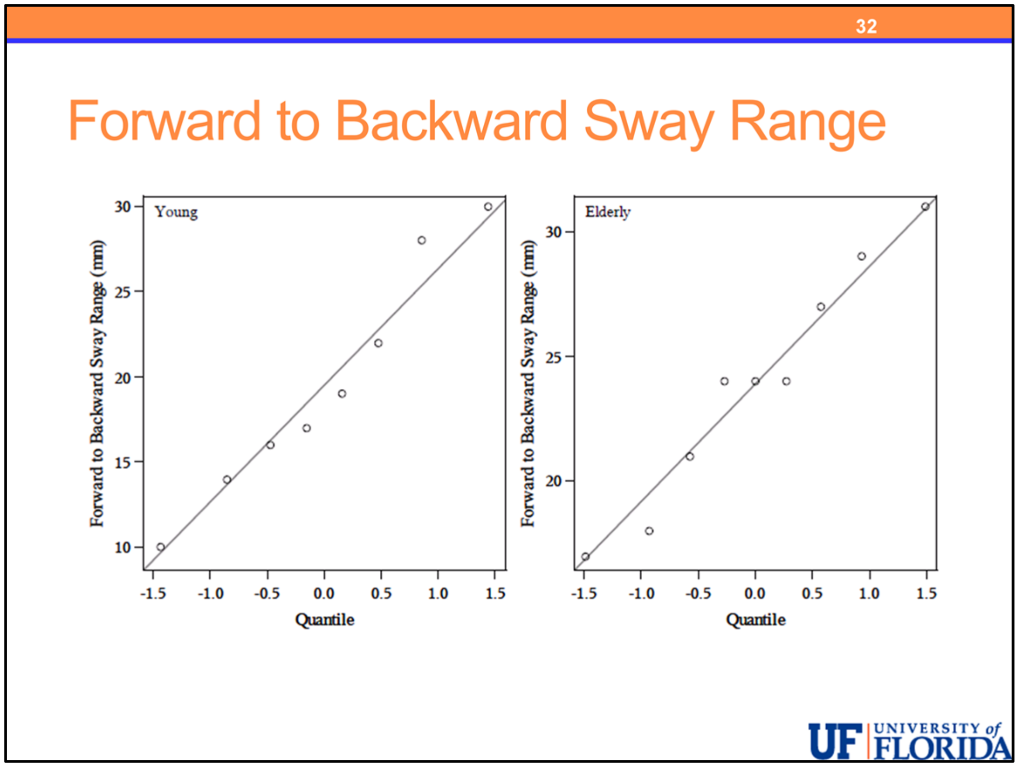
Now, boxplots for forward to backward sway range from SPSS. There seems to be some difference in the variation but, as this is a very small sample size, possibly this could be due to chance.

It does seem that the mean and median forward to backward sway range for elderly individuals is larger than that for young individuals but again, the sample size is small.



Here is the output from SAS when conducting the two-sample t-test.

Both distributions seem reasonably normal comparing the densities (solid vs. dotted line) on these histograms. We also see the boxplots again, in this case horizontally, under the histograms. There are no outliers in the data.



These are the QQ-plots from SAS which also show no reason for concern regarding the normality assumption.

Forward to Backward Sway Range

Age	Method	Mean	95% CL Mean		Std Dev	95% CL Std Dev	
Young		19.5000	13.7772	25.2228	6.8452	4.5259	13.9319
Elderly		23.8889	20.2744	27.5034	4.7022	3.1762	9.0084
Diff (1-2)	Pooled	-4.3889	-10.3977	1.6199	5.8017	4.2857	8.9792
Diff (1-2)	Satterthwaite	-4.3889	-10.6586	1.8808			

Method	Variances	DF	t Value	Pr > |t|
Pooled	Equal	15	-1.56	0.1404
Satterthwaite	Unequal	12.222	-1.52	0.1534

Equality of Variances				
Method	Num DF	Den DF	F Value	Pr > F
Folded F	7	8	2.12	0.3144

$$-10.4 < Y - E < 1.62 \rightarrow E - 10.4 < Y < E + 1.62$$

UF UNIVERSITY of FLORIDA

Now we move into the output for the Two-sample T-test on forward to backward sway range between young and elderly patients.

Notice in SAS the output lists young and then elderly. This indicates that the SAS output will be estimating mu-sub_Young minus mu-sub-Elderly.

For SAS we begin by looking for the p-value of the test for equality of variances, which is 0.3144, outlined in the lower right of this output.

Thus we fail to reject the null hypothesis that the variances are equal and so we can use the equal variances row in the tables, also outlined.

We find a p-value for the equal variances two sample t-test of 0.1404 and so there is not enough evidence to conclude that the population mean ***forward to backward sway range*** differs between young and elderly individuals.

The appropriate 95% confidence interval for the difference between the population mean for young and that for elderly is given as -10.4 to 1.62.

We can interpret our estimate and confidence interval as follows.

Based upon this study, we estimate that the mean forward to backward sway range for young individuals is 4.4 mm less than that for elderly individuals. However, the 95%

confidence interval indicates that the mean for young individuals could be as much as 10.4 mm less to as much as 1.62 mm MORE than that for elderly individuals.

Plausible values for the true mean difference (young – elderly) range from large negative values to small positive values and include the possibility that the true mean difference could be zero.

34

		Levene's Test for Equality of Variances	
		F	Sig.
Forward to Backward Sway Range (mm)	Equal variances assumed	1.383	.258
	Equal variances not assumed		

	Age Group
Forward to Backward Sway Range (mm)	Elderly
	Young
Side to Side Sway Range (mm)	Elderly
	Young

		t-test for Equality of Means		
		t	df	Sig. (2-tailed)
Forward to Backward Sway Range (mm)	Equal variances assumed	1.557	15	.140
	Equal variances not assumed	1.522	12.222	.153

		t-test for Equality of Means			
		Mean Difference	Std. Error Difference	95% Confidence Interval of the Difference	
				Lower	Upper
Forward to Backward Sway Range (mm)	Equal variances assumed	4.389	2.819	-1.620	10.398
	Equal variances not assumed	4.389	2.883	-1.881	10.659

-1.62 < E – Y < 10.4 → Y – 1.62 < E < Y + 10.4

UF UNIVERSITY of FLORIDA

In SPSS, we have the reverse order for our comparison, elderly – young. So our test statistic, mean difference, and confidence interval values are all reversed. Otherwise, the results are equivalent.

For SPSS we begin by looking for the p-value of the test for equality of variances, which is 0.258, outlined in the right column of the first table of this output. Notice the p-value is different from SAS and indeed the test used by SPSS may be preferred as it is less sensitive to outliers and departures from normality. It is possible that SAS users and SPSS users may get different results for this test and thus choose a different row for their t-test.

In this case we get the same conclusion as for SAS by failing to reject the null hypothesis that the variances are equal and so we would still use the equal variances row in the tables outlined in the output.

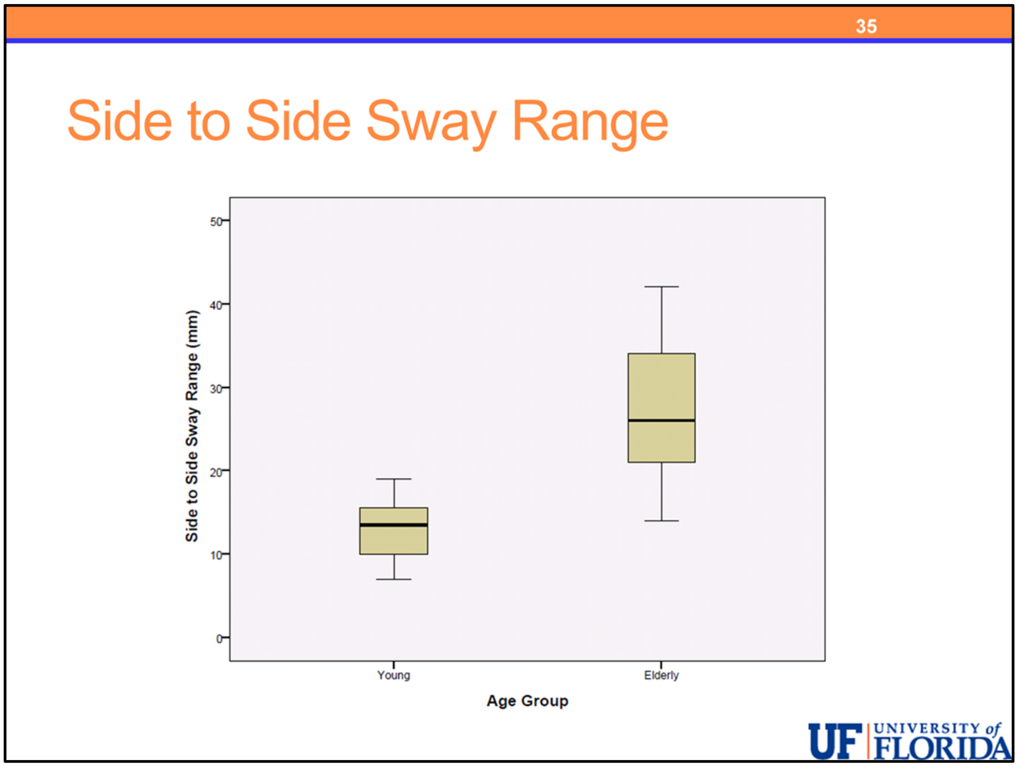
We find a p-value for the equal variances two sample t-test of 0.140 and so there is not enough evidence to conclude that the population mean ***forward to backward sway range*** differs between elderly and young and individuals.

The appropriate 95% confidence interval for the difference between the population mean for elderly and that for young is given as -1.62 to 10.4.

We can interpret our estimate and confidence interval as follows.

Based upon this study, we estimate that the mean forward to backward sway range for elderly individuals is 4.4 mm greater than that for young individuals. However, the 95% confidence interval indicates that the mean for elderly individuals could be as much as 1.62 mm less than to as much as 10.4 mm more than that for young individuals.

Plausible values for the true mean difference (elderly – young) range from small negative values to large positive values and include the possibility that the true mean difference could be zero.

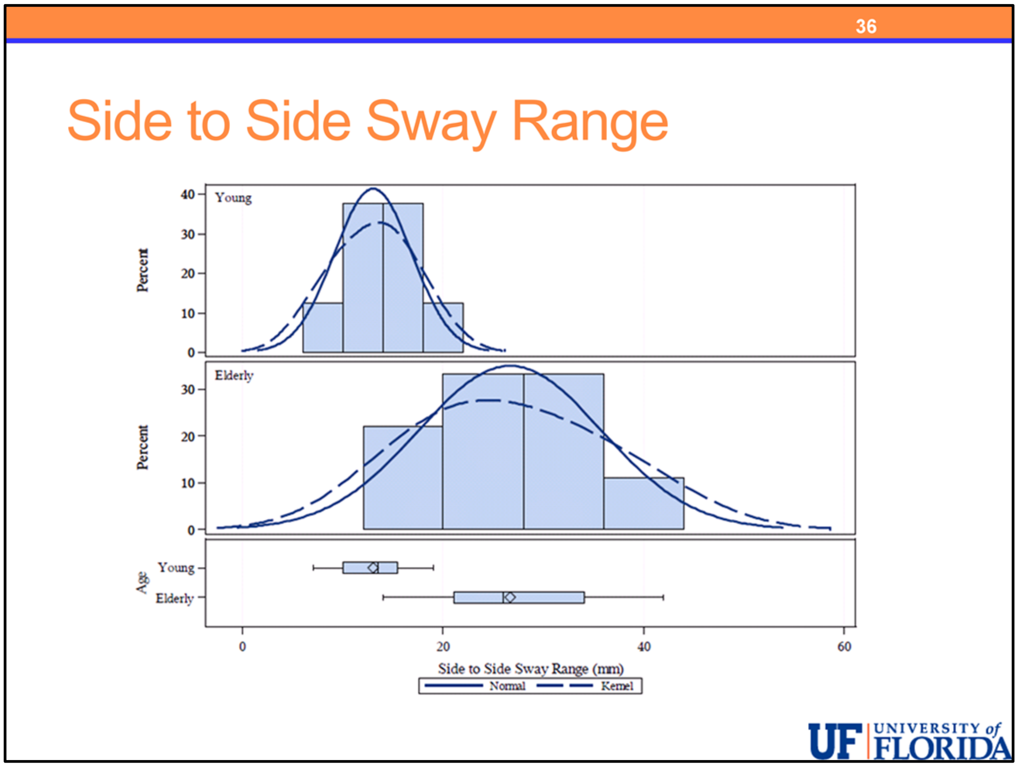


Now for Side to Side Sway Range.

The boxplots show a much larger difference in variation with the distribution of young individuals having a much smaller spread than that for elderly individuals.

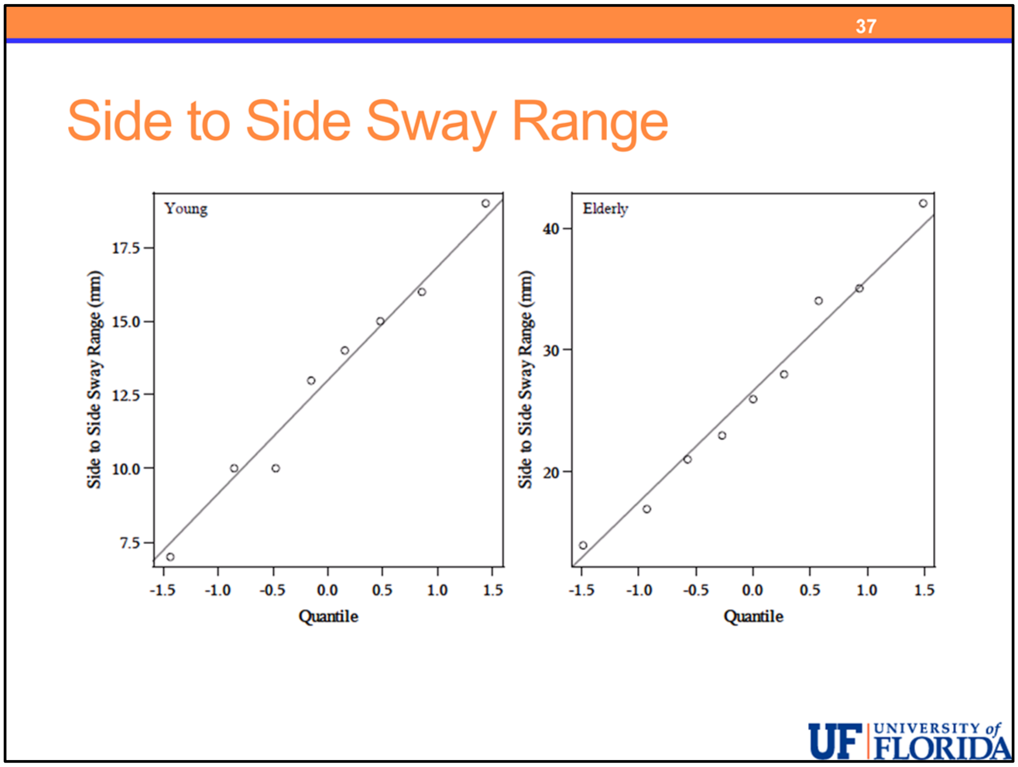
It does seem a more obvious difference exists for side-to-side sway.

Elderly individuals tend to have larger side to side sway than young individuals.



Here is the output from SAS when conducting the two-sample t-test.

Both distributions seem reasonably normal comparing the densities (solid vs. dotted line) on these histograms. We also see the boxplots and there are no outliers in the data.



These are the QQ-plots from SAS which also show no reason for concern regarding the normality assumption.

38

Side to Side Sway Range

Age	Method	Mean	95% CL Mean		Std Dev	95% CL Std Dev	
Young		13.0000	9.7776	16.2224	3.8545	2.5485	7.8449
Elderly		26.6667	19.6849	33.6484	9.0830	6.1351	17.4009
Diff (1-2)	Pooled	-13.6667	-21.0582	-6.2751	7.1368	5.2720	11.0455
Diff (1-2)	Satterthwaite	-13.6667	-20.9703	-6.3631			

Method	Variances	DF	t Value	Pr > |t|
Pooled	Equal	15	-3.94	0.0013
Satterthwaite	Unequal	11.052	-4.12	0.0017

Equality of Variances				
Method	Num DF	Den DF	F Value	Pr > F
Folded F	8	7	5.55	0.0358

UF UNIVERSITY of FLORIDA

In the SAS output, we begin by looking for the p-value of the test for equality of variances, which is 0.0358.

Thus here we do reject the null hypothesis that the variances are equal and so we can NOT use the equal variances row in the tables, we should instead use the unequal variances row outlined in the output.

We find a p-value for the unequal variances two sample t-test of 0.0017 and so there is enough evidence to conclude that the population mean ***side to side sway range*** differs between young and elderly individuals.

The appropriate 95% confidence interval for the difference between the population mean for young and that for elderly is given as -20.97 to -6.36.

We can interpret our estimate and confidence interval as follows.

Based upon this study, we estimate that the mean side to side sway range for young individuals is 13.7 mm less than that for elderly individuals. However, the 95% confidence interval indicates that the mean for young individuals could be as little as 6.36 mm to as much as 20.97 mm less than that for elderly individuals.

Plausible values for the true mean difference (young – elderly) are all negative and hence zero is not a plausible value.

39

		Levene's Test for Equality of Variances	
		F	Sig.
Side to Side Sway Range (mm)	Equal variances assumed	4.894	.043
	Equal variances not assumed		

		t-test for Equality of Means		
		t	df	Sig. (2-tailed)
Side to Side Sway Range (mm)	Equal variances assumed	3.941	15	.001
	Equal variances not assumed	4.116	11.052	.002

		t-test for Equality of Means			
		Mean Difference	Std. Error Difference	95% Confidence Interval of the Difference	
				Lower	Upper
Side to Side Sway Range (mm)	Equal variances assumed	13.667	3.468	6.275	21.058
	Equal variances not assumed	13.667	3.320	6.363	20.970

UF UNIVERSITY of FLORIDA

Once again the results in SPSS are reversed and yet reveal the same conclusion. We begin by looking for the p-value of the test for equality of variances, which is 0.043.

Thus here we do reject the null hypothesis that the variances are equal and so we can NOT use the equal variances row in the tables, we should use the unequal variances row - outlined in the output.

We find a p-value for the unequal variances two sample t-test of 0.002 and so there is enough evidence to conclude that the population mean ***side to side sway range*** differs between young and elderly individuals.

The appropriate 95% confidence interval for the difference between the population mean for elderly and that for young is given as 6.36 to 20.97.

We can interpret our estimate and confidence interval as follows.

Based upon this study, we estimate that the mean side to side sway range for elderly individuals is 13.7 mm more than that for young individuals. However, the 95% confidence interval indicates that the mean for elderly individuals could be as little as 6.36 mm to as much as 20.97 mm more than that for young individuals.

Plausible values for the true mean difference (elderly – young) are all positive and hence zero is not a plausible value.

Non-Parametric Tests

Hypothesis Test Summary

	Null Hypothesis	Test	Sig.	Decision
1	The distribution of Forward to Backward Sway Range (mm) is the same across categories of Age Group.	Independent-Samples Mann-Whitney U Test	.139[1]	Retain the null hypothesis.
2	The distribution of Side to Side Sway Range (mm) is the same across categories of Age Group.	Independent-Samples Mann-Whitney U Test	.001[1]	Reject the null hypothesis.

Asymptotic significances are displayed. The significance level is .05.

[1] Exact significance is displayed for this test.

We would get the same conclusions from the non-parametric Wilcoxon Rank-Sum test. The SPSS results are shown here with a p-value for forward to backward of 0.139 and one for side to side of 0.001.

41

Wilcoxon Scores (Rank Sums) for Variable f_b Classified by Variable Age					
Age	N	Sum of Scores	Expected Under H0	Std Dev Under H0	Mean Score
Elderly	9	96.50	81.0	10.360417	10.722222
Young	8	56.50	72.0	10.360417	7.062500
Average scores were used for ties.					

Wilcoxon Scores (Rank Sums) for Variable s_s Classified by Variable Age					
Age	N	Sum of Scores	Expected Under H0	Std Dev Under H0	Mean Score
Elderly	9	112.50	81.0	10.379561	12.50000
Young	8	40.50	72.0	10.379561	5.06250
Average scores were used for ties.					

Wilcoxon Two-Sample Test	
Statistic	56.5000
Normal Approximation	
Z	-1.4478
One-Sided Pr < Z	0.0738
Two-Sided Pr > |Z|	0.1477
t Approximation	
One-Sided Pr < Z	0.0835
Two-Sided Pr > |Z|	0.1670
Z includes a continuity correction of 0.5.	

Wilcoxon Two-Sample Test	
Statistic	40.5000
Normal Approximation	
Z	-2.9866
One-Sided Pr < Z	0.0014
Two-Sided Pr > |Z|	0.0028
t Approximation	
One-Sided Pr < Z	0.0044
Two-Sided Pr > |Z|	0.0087
Z includes a continuity correction of 0.5.	

The SAS results are more complex. The two-sided p-values for either the Z or t approximation are acceptable.

For forward to backward on the left, we find a p-value of 0.1477 for the Z or 0.1670 for the t.

And for side to side on the right, we find a p-value of 0.0028 for the Z or 0.0087 for the t.

Finally, for our test involving forward to backward sway range, since we failed to reject the null hypothesis, it is possible that we could have made a type II error.

In context we would not conclude that there is a difference in the mean forward to backward sway when in fact there is a difference.

And for our test involving side to side sway range, since we rejected the null hypothesis, it is possible that we could have made a type I error.

In context we would conclude that there is a difference in the mean side to side sway when in fact there is NOT a difference.