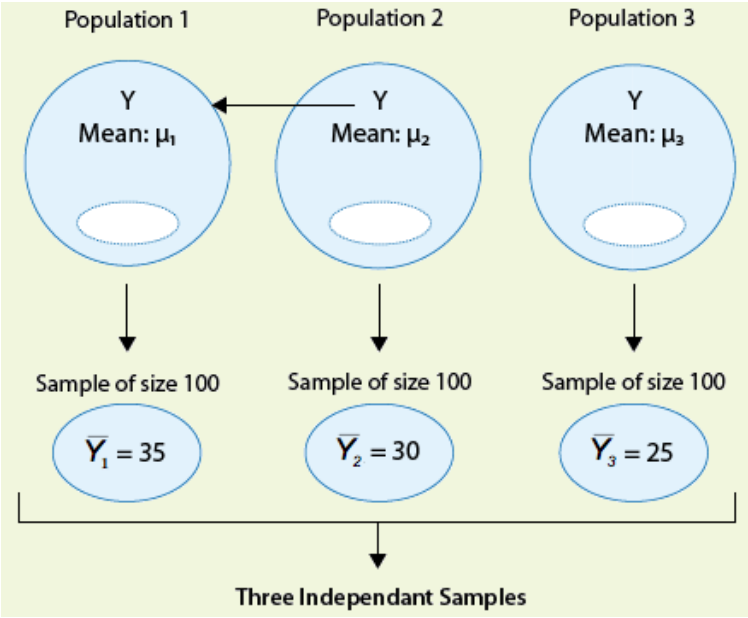
Learn By Doing #1 – Idea of One-Way ANOVA

Consider the following generic situation:

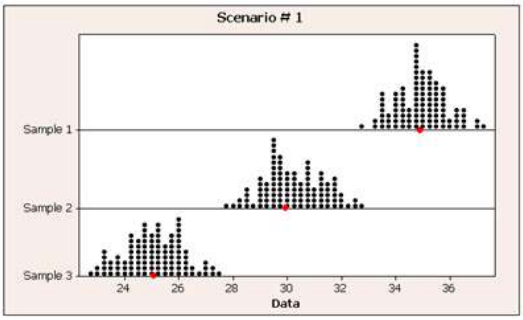


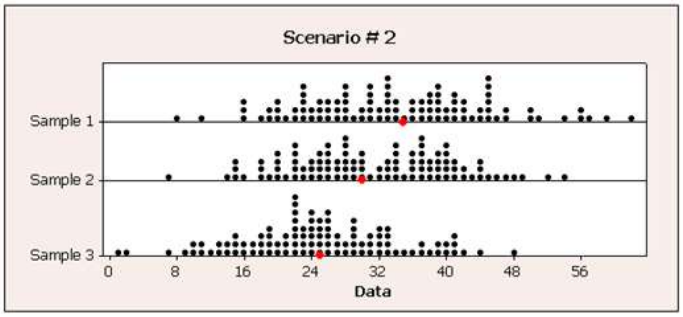
In this case, we are testing

H_0: $\mu_1 = \mu_2 = \mu_3$

H_a: Not all the μ's are equal

The following are two possible scenarios of the data (note in both scenarios the sample means are 25, 30, and 35).





Learn By Doing #1 – Idea of One-Way ANOVA – continued

Consider Scenario 1 (compared with Scenario 2):

Choose the correct response below.

- **Within-group variation is relatively small, and therefore the F statistic will be relatively large, indicating that Ho will probably be rejected, and we'll conclude that the population means are not all equal.**

Correct. Indeed the within-group variation in Scenario 2 is larger than in Scenario 1.

The within-group variation appears in the denominator of the F statistic, and therefore if the within-group variation is relatively small (as it is for Scenario 1), the F statistic will be relatively large.

The larger the F statistic (which happens when the variation within groups is relatively small) the more evidence we have against H_o.

When H_o is rejected, we accept the alternative claim, which in the ANOVA F-test says that the means are not all equal.

- **Within-group variation is relatively large, and therefore the F statistic will be relatively small, indicating that Ho will probably not be rejected, and we'll not have enough evidence to conclude that the population means are not all equal.**

Incorrect. Look at the spread within each of the groups in Scenario 1 as opposed to Scenario 2. For example, in Sample 3 in Scenario 1, the data range from roughly 23 to 27.5, while in Scenario 2, the data in Sample 3 range from 1 to 48.

Note that the within-group variation appears in the denominator of the F statistic, and therefore if the within-group variation is relatively small (as it is for Scenario 1), the F statistic will be relatively large.

The larger the F statistic (which happens when the variation within groups is relatively small) the more evidence we have against Ho.

When Ho is rejected, we accept the alternative claim, which in the ANOVA F-test says that the means are not all equal.

Learn By Doing #1 – Idea of One-Way ANOVA – continued

Now consider Scenario 2 (compared to Scenario 1):

Choose the correct response below.

- **Within-group variation is relatively small, and therefore the F statistic will be relatively large, indicating that Ho will probably be rejected, and we'll conclude that the population means are not all equal.**

Incorrect. Look at the spread within each of the groups in Scenario 2 as opposed to Scenario 1. For example, in Sample 3 in Scenario 2 the data range from 1 to 48, while in Scenario 1, the data in Sample 3 range only from roughly 23 to 27.5.

Note that the within-group variation appears in the denominator of the F statistic, and therefore if the within-group variation is relatively large (as it is for Scenario 2), the F statistic will be relatively small.

The smaller the F statistic (which happens when the variation within groups is relatively large) the less evidence we have against Ho.

When we cannot reject Ho, we're essentially saying that it could be that the means are equal. Recall, however, that not rejecting Ho does not mean that we accept it. In hypothesis testing, we never accept Ho.

- **Within-group variation is relatively large, and therefore the F statistic will be relatively small, indicating that Ho will probably not be rejected, and we'll not have enough evidence to conclude that the population means are not all equal.**

Correct. Indeed, the within-group variation in Scenario 1 is smaller than in Scenario 2.

The within-group variation appears in the denominator of the F statistic, and therefore if the within-group variation is relatively large (as it is for Scenario 2), the F statistic will be relatively small.

The smaller the F statistic (which happens when the variation within groups is relatively large) the less evidence we have against Ho.

When we cannot reject Ho, we're essentially saying that it is possible that the means are equal. Recall, however, that not rejecting Ho does not mean that we accept it. In hypothesis testing, we never accept Ho.

Learn By Doing #2 – Idea of One-Way ANOVA

Suppose that we would like to compare four populations (for example, four races/ethnicities or four age groups) with respect to a certain psychological test score. More specifically we would like to test

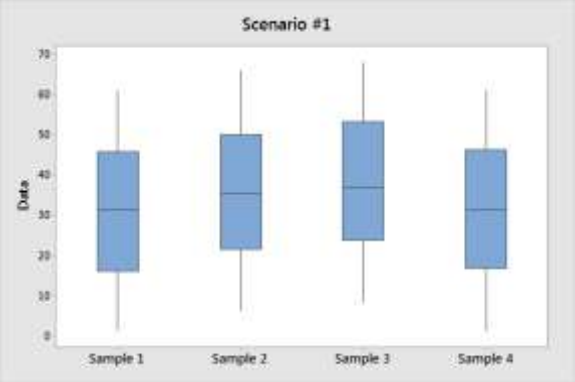
H_0: $\mu_1 = \mu_2 = \mu_3 = \mu_3$

H_a: Not all the μ's are equal

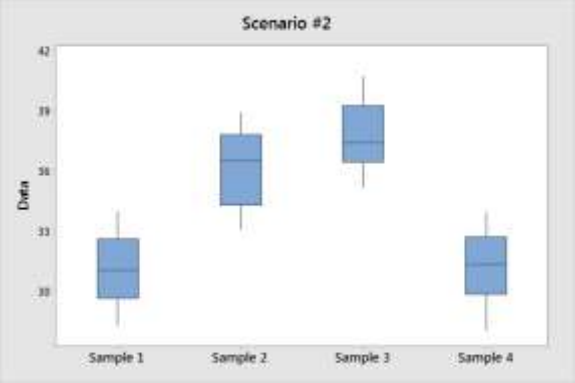
where μ_1 is the mean test score in population 1, μ_2 is the mean test score in population 2, μ_3 is the mean test score in population 3, and μ_4 is the mean test score in population 4.

We take a random sample from each population and use these four independent samples in order to carry out the test.

The following are two possible scenarios for the data:



	Score Averages
Sample 1	$\bar{x}_1 = 30.9$
Sample 2	$\bar{x}_2 = 35.6$
Sample 3	$\bar{x}_3 = 38.0$
Sample 4	$\bar{x}_4 = 31.2$



	Score Averages
Sample 1	$\bar{x}_1 = 31.1$
Sample 2	$\bar{x}_2 = 36.2$
Sample 3	$\bar{x}_3 = 37.7$
Sample 4	$\bar{x}_4 = 31.2$

Note that in both scenarios, the score averages of the four samples are very similar.

Learn By Doing #2 – Idea of One-Way ANOVA – continued

Question 1:

In which scenario do the data have a larger within-group variability?

HINT

Look at the two sets of boxplots. In which scenario do we see more spread within each of the groups? Look at the IQR (represented by the height of the box) as well as the full range.

Scenario 1.

Correct. In scenario 1 we see more spread within each of the four samples (IQR roughly 30 and full range roughly 60) compared to scenario 2 (IQR roughly 3 and the full range is roughly 6).

Scenario 2.

Incorrect. What is the approximate IQR in each of the samples in scenario 1? What is the approximate IQR in each of the samples in scenario 2? Which one is larger?

Both scenarios have the same within-group variability.

Incorrect. The two scenarios do not have the same within-group variability. What is the approximate IQR in each of the samples in scenario 1? What is the approximate IQR in each of the samples in scenario 2? Which one is larger?

Learn By Doing #2 – Idea of One-Way ANOVA – continued

Question 2:

In which scenario do the data have a larger F test statistic?

HINT

Recall that the F test statistic = (Variation between sample means)/(Variation within groups) and that in both scenarios the sample means are very similar and so the numerator in both scenarios is practically the same.

Scenario 1.

Incorrect. Recall that in scenario 1 we have a larger within group variability. In other words, in scenario 1 the denominator of the F test statistic is larger. Recall also that in both scenarios the numerator of the F test statistic is roughly equal (since the sample means in both scenarios are very similar).

Scenario 2.

Correct. In both scenarios the numerator of the F test statistic is roughly equal (since the sample means in both scenarios are very similar). Since in scenario 2 we have smaller within group variability and therefore the denominator of the F test statistic is smaller, the F test statistic is larger.

Both scenarios have the same F test statistic.

Incorrect. Recall that in scenario 1 we have a larger within group variability. In other words, in scenario 1 the denominator of the F test statistic is larger. Recall also that in both scenarios the numerator of the F test statistic is roughly equal (since the sample means in both scenarios are very similar).

Learn By Doing #2 – Idea of One-Way ANOVA – continued

Question 3:

In which scenario are we more likely to reject H0 and conclude that the four population score means are not all equal?

HINT

Recall that the larger the F test statistic, the more evidence the data provide against the null hypothesis.

Scenario 1.

Incorrect. Scenario 2 has smaller within-group variability and therefore a larger F test statistic. Recall that the larger the F test statistic, the more evidence the data provide against the null hypothesis.

Scenario 2.

Correct. Scenario 2 has smaller within-group variability and therefore a larger F test statistic. The larger the F test statistic, the more evidence the data provide against the null hypothesis.